Sustainability of Healthcare Information Exchanges: A Game-Theoretic Approach

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Based on our interactions with the key personnel of three different healthcare information exchange (HIE) providers in Texas, we study a setting where the HIE provider can offer value-added services in addition to base HIE connectivity. We find that costs related to maintaining the HIE and offering value-added services affect whether the HIE network will be established or not, whereas heterogeneity among healthcare practitioners (HPs) influence the participation levels in these networks. We find that in different circumstances, low-gain HPs choose not to join to HIEs. Hence, in order to entice more participation in HIEs, it might be beneficial for the policy-makers and/or HIE providers to (i) establish a second HIE in the region, (ii) propose more value to the low-gain HPs, or (iii) offer and/or incentivize value-added services. Contrary to intuition, we also find that an increase in the highest benefit HPs can get from the HIE can even decrease the number of participating HPs in the HIE. Besides, since the amount of funds from the government and the other agencies often change and will eventually cease, we analyze how the changes in the benefit HPs obtain from the HIE affect (i) participation in the network, (ii) HIE subscription fee and the fee for value-added service, (iii) number of HPs that request value-added service, and (iv) the net values of the HIE provider and HPs. We also present several other insights and provide guidelines for the policy-makers that may help them improve the sustainability of HIEs and increase the participation levels in these networks.

Key words: Healthcare management, HIE networks, game theory.

“The promise of health information exchanges is not in question, but whether they can be sustained and thrived is.”

– Douglas Page, Hospitals and Health Networks’s Digital Magazine (Page 2010)

1. Introduction

Healthcare spending in U.S. is increasing rapidly. In 2012, it grew 3.7 percent to reach $2.8 trillion or around $9,000 per person (CMS 2014). More importantly, the healthcare spending accounted for 17.2 percent of nation’s gross domestic product (GDP) in 2012. Therefore, the national effort on healthcare is becoming more pronounced in order to reduce costs and increase the quality of
the system (Menon et al. 2000, Agarwal et al. 2010b, Aron et al. 2011, Menon and Kohli 2013). There are numerous reasons for high healthcare spending such as overuse and misuse of diagnostic testing and emergency department services, avoidable hospitalization and re-hospitalization, and fragmented information infrastructure or technology that support patient care (Bohmer 2011, Weinberger 2011, Mishra et al. 2012). In this paper, we focus and study the dynamics of healthcare information exchange (HIE) networks that relieve the problems due to the fragmentation of patient healthcare records or information. Patient health records are fragmented because patients tend to move from a healthcare practitioner (HP) to another, but their records do not move. Therefore, the U.S. government is incentivizing HIEs in order to connect HPs effectively that can help the nation save around $80 billion (Walker et al. 2005).

Because of the inherent benefits of HIE networks, spending to establish such networks is increasing dramatically. A Black Book HIE survey estimates that the industry spending will triple by 2014 (Yahoo News 2012). In this survey, 84% of the executives indicated that they are actively discussing regional alignment and solution purchases to address stakeholder interoperability — in particular HIEs. Despite their promise, HIEs are still far from meeting the expectations (Agarwal et al. 2010a). Some of the barriers that hinder extracting the full potential of HIE services are willingness to share information or privacy concerns (Angst and Agarwal 2009, Anderson and Agarwal 2011), adoption and network effects (Agarwal et al. 2010b, Romanow et al. 2012, Yaraghi et al. 2013, Yaraghi et al. 2014), problems in governmental regulations (Ozdemir et al. 2011), and, maybe more importantly, sustainability and other financial factors (Fontaine et al. 2010, Terry 2013). The growth and the success of these HIE networks depend heavily on the decisions taken by the HIE providers and the HPs (Walker et al. 2005). Agarwal et al. (2010c) argue that the longevity of HIEs is at question without government support, and they state that offering value-added services is one of the possible business strategies that can help sustain a healthy HIE. Hence, HIEs around the U.S. focus on how to financially sustain their operations (Hall 2013).
In this paper, we analyze the sustainability of the HIEs and derive the equilibrium behaviors of an HIE provider and the HPs in a game-theoretic setting. Our proposed model is primarily based on our interactions with a number of HIEs in Texas (Calhoun 2013, Samuels 2013, Smitherman 2013). In the next subsection, we briefly discuss some preliminaries and the background of HIEs.

1.1. Preliminaries and the Background of HIEs

HIEs can be defined as information sharing technologies or mechanisms that automate the transfer and sharing of health-related information typically stored in multiple organizations, while maintaining the context and integrity of the information being exchanged (HIMSS 2009a). An HIE provides access to and retrieval of patient information to authorized users in order to provide safe, efficient, effective, and timely patient care. HIEs are typically formed by a group of participants from a specific area or a region. A true HIE involves multi-directional flows of information electronically among HPs such as hospitals, physicians, clinics, labs, etc. Furthermore, an HIE is not only about moving clinical information to the right place — it also affects the clinical workflows by making the data available to doctors and nurses when they need them to make decisions. Thus, it provides improved patient safety by sharing their medical data. In addition, cost reductions due to HIEs include elimination of duplicate tests, recovery of missing patient health data, elimination of paper, ink, and manual document printing, and reduction of phone calls and follow-ups with labs for test results. An HIE assures a strong chain of custody of patient data and their movements. It also helps in providing accurate feedback to public health registries (Fontaine et al. 2010). Finally, participating in HIEs is a requirement for HPs to receive stimulus funds (Page 2010).

The HIEs got their start in the early 1990s when the healthcare industry set up community health information networks to share patient medical records among HPs (see Figure 1). These networks were encumbered by pre-Internet connectivity costs and did not live up to expectations (Lorenzi 2003). Some, however, have endured and are still in operation, such as the Indiana Health Information Exchange (IHIE), the Utah Health Information Network (UHIN), and the HealthBridge
in Cincinnati. The current era of HIEs began in earnest in 2004 with the creation of the Office of National Coordinator (ONC) by the Department of Health and Human Services (HHS). Since then, as Figure 1 illustrates, HIE development has been encouraged by the federal government with many nationwide projects like Nationwide Health Information Network (NHIN) or Nationwide Health Information Network (NwHIN) that has since taken the name eHealth Exchange, Meaningful Use Incentives program from Center of Medicare and Medicaid Services (CMS), Regional Extension Center (REC) program, and State-wide HIE Cooperative Agreement program (Kolkman et al. 2011). According to the eHealth Initiative report, there were a dozen of HIE initiatives in 2004, but this number grew to approximately 255 by 2011. More importantly, this number is expected to grow significantly after 2012 (Yahoo News 2012). Next, we discuss the research questions that we study in this paper along with our contributions.
1.2. Research Questions and Contributions

Substantial progress is being made to make HIEs a reality that would benefit the patients and HPs (AHRQ 2006). However, challenges and barriers remain — most notably funding and sustainability (HIMSS 2009b, Agarwal et al. 2010c, Sridhar et al. 2012). Hence, HIEs in the U.S. need to focus on financially sustaining their operations (Hall 2013). In general, HIEs can be not-for-profit, public utility, physician collaborative, or for-profit (Berry and Johnson 2012). However, in order to remain or become sustainable, especially after the governmental funds cease, all HIEs strive or will have to strive for maximizing their profits. Therefore, independent of their nature, we regard the HIEs as profit maximizing organizations in our research.

Most of the well-established HIEs and some of the newly established HIEs offer value-added services. Hence, our main model enables us to examine the dynamics of an HIE network in which the HIE provider offers value-added services in addition to the base HIE connectivity. We also consider a special case of our model (referred to as the benchmark case), where we assume that the HIE provider does not offer value-added services but just the base HIE connectivity. This model represents business practices of most of the newly established HIEs since they usually do not offer value-added services. In our model (and in its benchmark case), we use a game-theoretic framework in order to derive how the HIE provider should set the membership fee and the characteristics of HPs that join the HIE. Based on the results of our model, we first analyze the following question: when should the HIE provider establish the network? We find in the benchmark case that the HIE provider should establish the network only if at least half of the HPs in the region find it beneficial to participate. We also show that the cost of accommodating each HP in the HIE has the most important role in determining whether an HIE should be established in our model (and in its benchmark case).

The next question we analyze is following: what factors impact the extent of participation and what could be done to increase the participation level? Our results show that if the cost of accommodating each HP in the HIE is not prohibitive or if there is enough government support, the
heterogeneity among the HPs (in terms of their expected benefit from the HIE membership) determines the extent of participation. We find that some low-gain HPs choose not to join the HIE if there are more high-gain HPs in the region.\textsuperscript{1} In these circumstances, policy-makers and the HIE provider could employ different strategies to encourage participation. Firstly, the minimum benefit the HPs can obtain from joining the HIE could be increased. For example, if small HPs do not have functional electronic health record (EHR) systems, HIE providers might offer EHR applications for free or at a very low price. In the U.S., several HIE providers are currently offering this incentive. However, we find that these incentives cannot entice all HPs to join the network if the heterogeneity among the HPs is high. Interestingly, as the maximum benefit the HPs can obtain from the HIE increases, participation in the HIE might be lower in some cases. The second possible strategy for increasing the participation is establishing more than one HIE. In some scenarios, establishing more than one HIE in the same region (where one of them focuses on low-gain HPs) entices more HPs to join. Hence, there might be an incentive for policy-makers to establish more than one HIE in the same region. The third option for the HIE providers is to offer HIE related value-added services. Offering value-added services is the main difference between our main model and its benchmark case. Hence, comparing them provides several useful insights regarding the benefits and management of value-added services. In particular, we derive how the HIE provider decides on the level of the value-added service and its fee, and which HPs request this service. This analysis also helps us outline the conditions under which the network expands because of the value-added service.

The amount of funds from the government and the other agencies often change and will eventually cease. Therefore, the third question that we analyze is the following: how does the changes in the benefit HPs obtain from the HIE affect (i) participation in the network, (ii) HIE subscription fee and the fee for value-added services, (iii) number of HPs that request value-added service, and (iv)

\textsuperscript{1} For simplicity, we refer to the HPs that obtain low benefits from joining the HIE as low-gain HPs. Similarly, the high-gain HPs represent those HPs that obtain high benefits from joining the HIE.
the net values of the HIE provider and HPs? In addition, we present several other insights that may be useful for the HIE provider, the HPs, and the policy-makers. In the next subsection, we review the related literature and highlight our contributions with respect to past studies.

1.3. Literature Review

Our study is broadly related to two streams of literature: (i) HIE research, and (ii) network externalities/platform adoption. In the last few years, several case studies (Fontaine et al. 2010, Feldman et al. 2013), progress reports (Agarwal et al. 2010c), white papers (Covich et al. 2011), and surveys (Patel et al. 2012, Yahoo News 2012) have been published regarding different aspects of HIEs. As discussed in these studies, key benefits for HPs from joining an HIE include facilitation of compliance with state and federal mandates, cost savings, and improved quality of patient care (Fontaine et al. 2010, Joshi 2010). The HIE literature further discusses how the values gained from the HIE can be converted to a sustainable business model for HIE providers (Marchibroda 2007). In this paper, we analyze whether the benefits of joining the HIE outweigh the costs involved in a setting with multiple HPs. We also analyze the effect of value-added services on the participation in the HIE network and on the net values of the HIE provider and the HPs. We formulate our model based on the HIE literature and our interactions with different HIE providers.

There are two other streams of research in the HIE literature. The first stream is related to the HIE policy. HIE policies are dependent on government requirements for different types of reporting, such as those required by CMS. Interested readers can refer to Marchibroda (2007), Adler-Milstein et al. (2011), and Adjerid et al. (2011) for details. The second HIE research stream studies the technical aspect of networks, and is not directly related to our study. Interested readers can refer to the representative papers by Lorenzi (2003) and Vest and Jasperson (2010).

Network effect or network externality implies that the value of a product or service (sometimes through adoption of a platform) depends on the total number of users. For an extensive review of network effects and competition, see Farrell and Klemperer (2007). Literature on network exter-
nalities and platform adoption focuses on switching costs, lock-in effects, or competition among different platforms each of which displays network effects. In this literature, users can usually adopt only one network out of all networks. Hence, the focus is on (i) which platform or network product dominates, (ii) and whether competition allows excess profits to network owners or increases values of consumers. In our paper, however, HIE membership and value-added services are two simultaneous network externalities. Also, unlike most of the past studies, our focus is on participation levels in the network under different conditions.

Further, in classic network externality models, the network externality is equally valuable to all customers. In particular, customers are assumed to have homogeneous network valuations although they have heterogeneous product valuations (Fudenberg and Tirole 2000, Jing 2007, Prasad et al. 2010, Niculescu et al. 2012). In some models, product valuations and the benefits customers obtain from the network are correlated (e.g., Sundararajan 2003, 2004). In particular, a customer with a high (resp., low) product or service valuation also has a high (resp., low) valuation for the network. Our paper is similar to the latter group of papers since the types of HPs determine both their valuation of the HIE membership and the valuation of the HIE-related value-added services. However, our paper is unique in the sense that we have two simultaneous network externalities, in particular one because of base HIE services and the other because of HIE related value-added services. In addition, the membership to the value-added service network requires membership for the HIE network. Another different aspect of our model is that the HIE provider can determine the level of value-added service it will offer to its participants. To the best of our knowledge, none of the past studies analyzes two simultaneous network externalities where one of them requires membership to the other, and the optimization of the level of service offered to the participants in the second network.

The rest of the paper is organized as follows. In Section 2, we present the details of two representative HIEs that form the underpinnings of our model and its benchmark case. Next, in Section 3, we present the formulation of our model. In Section 4, we present the solutions of our model and its
benchmark, and provide useful managerial insights. Finally, Section 5 summarizes the implications for HIE providers, HPs, and the policy-makers.

2. HIE Scenarios

We present our main model and its benchmark case in Section 3. In formulating our model, we worked with several HIE providers in order to understand the critical issues involved in their sustainability. In our main model, the HIE provider offers value-added services related to HIE in addition to the base HIE service. This business practice is prevalent among well established HIEs, as in the case of Integrated Care Collaboration (ICC) and Critical Connection that are based in Texas. In the benchmark case, the HIE provider offers the base HIE service, but not the value-added services. Most of the newly established HIE providers follow this practice and focus only on base service, as in the case of a start-up HIE Southeast Texas Health Systems (SETHS). For our modeling purpose, ICC and Critical Connection are similar in nature, and therefore we provide the details of only ICC and SETHS. Although our model is developed in the context of these HIEs, many HIE providers in the nation either have plans for or already offer HIE related value-added services. However, some of the HIE providers focus only on the base HIE services (especially, in the case of newly established HIEs), and therefore we study this business setting in the benchmark case in Section 3.2. Hence, our model is generic, and the results as well as insights are applicable to most of the HIE settings.

2.1. An HIE Provider that Offers Value-Added Services: ICC

Integrated Care Collaboration (ICC)\(^2\) is an established HIE, which is located in Central Texas dedicated to the collection, analysis, and sharing of health information. The primary objective of ICC is to create and operate a regional HIE that is sufficiently trusted and valued by the stakeholders to enable improved care coordination and a foundation for sustainability. ICC includes around 70 hospitals and 5600 physicians located in Central and Eastern Texas. ICC, established in 2002, uses a web-based community health record called ICare that stores patient demographic

\(^2\)http://icc-centex.org/health-information-exchange (Last Accessed: Nov. 24, 2014)
and encounter information. Recently, ICare 2.0 has been launched based on an open-source HIE technology that is accessible through a web-based portal. The redesigned ICare solution integrates a robust open source HIE platform that includes interface, integration, clinical data repository, and master patient index, with data warehousing solutions. The portal provides access to an aggregate patient medical record to the participating HPs. The ICare system is being further extended by custom Java and Web-based development efforts provided by Centex System Support Services that was formed to operate as the technology arm of the ICC. The combination of open source technologies and in-house development expertise not only reduces short-term and long-term development, maintenance, and operational costs, but also facilitates the ICC’s ability to offer additional services to the interested participating HPs in its network.

Most of the HIE organizations like ICC mature, they provide other value-added services in addition to the base HIE functionality (Agarwal et al. 2010c, Covich et al. 2011). HIEs need to maximize their net value in order to improve sustainability. They achieve this objective by strategically setting the fee for joining the HIE, price of the value-added service, and the level of the value-added service (Agarwal et al. 2010c, Samuels 2013, Smitherman 2013). In the next section, we briefly discuss a newly established HIE provider SETHS.

2.2. An HIE Provider that Focuses on Base Service: SETHS

The Southeast Texas Health Systems (SETHS)\(^3\) is a start-up HIE, which is a cooperation of rural hospitals in Southeast Texas. One of the main objectives of SETHS is to implement and sustain an HIE among healthcare practitioners in order to ultimately improve the health status of the population in Southeast Texas. SETHS’s plan is to develop and operate a high-quality, cost-effective healthcare data exchange system for small and rural HPs. Other goals include (i) supporting interoperability with the eHealth Exchange, (ii) supporting core components of an interoperable HIE, (iii) assisting in exchange of health and personal information from disparate

source systems, (iv) developing a permissions and messaging infrastructure that allows stakeholder access to data in compliance with all applicable privacy/security and standards regulations, (v) aiding the continuity of patient care by facilitating referrals and transitions of care between HPs, and finally (vi) aligning with the state-wide healthcare infrastructure in Texas. According to the Executive Director of SETHS, such an HIE will provide following benefits to its members: (i) create incentives to share data with other HPs, (ii) improve ability to negotiate reimbursements by demonstrating evidence-based clinical decision support, and (iii) provide capacity to develop protocols and goals for outcomes (Calhoun 2013).

The technical architecture of the HIE operated by SETHS (called SOPHIE) is based on an open-source HIE gateway called HIEOS that supports core HIE services for its members like patient management, record locator provision, repository management, cross community shared patients locator facility, and secured and encrypted communication. An application integration platform mechanism is also used to interface with EHR systems in member hospitals. SOPHIE currently does not have additional service offerings, such as e-prescribing or patient management tools. Rather, they focus only on base HIE service. This practice is common among most of the start-up HIEs (Calhoun 2013).

The start-up HIE providers, such as SETHS, need to maximize their net value in order to improve sustainability (Calhoun 2013). They achieve this objective by setting the subscription fee appropriately. A higher subscription fee increases the marginal revenue for the HIE provider, but it decreases the number of HPs that decide to join. Further, the more participants in the HIE, the more beneficial the HIE becomes to participating HPs because there is more information to share. Hence, the HIE provider needs to obtain the equilibrium subscription fee such that its net value is maximized considering the effect of network externality. As we discuss in detail later, the benchmark case presented in Section 3.2 represents the business practices of most startup HIEs like SETHS since they usually do not offer value-added services.
3. Model

In line with the first HIE scenario discussed in Section 2.1, here we consider that the HIE provider offers HIE related value-added services to the participating HPs if they request it. Value-added services can range from e-prescribing to patient management tools (Covich et al. 2011). We analyze the equilibrium behavior of HPs and the HIE provider in this environment. In particular, we are interested in the optimal level of value-added service that the HIE provider should offer, the equilibrium network size, the number of HPs that request the value-added service, the subscription fee of the base HIE service, the price of the value-added service, and the conditions that would help the value-added service increase the network size. We denote the total number of potential HPs that consider joining the HIE by $N$. These include rural hospitals, laboratories, family practice clinics, wellness centers, rehab centers, and federally qualified health centers.

3.1. Details of the Main Model

The net benefit of an HP from joining the HIE (that excludes the benefits from value-added services) is the difference between the gains and the costs of joining the HIE (adjusted to a month). As discussed earlier, different benefits of the base HIE service include improved patient safety, cost reductions due to elimination of duplicate tests, recovery of missing patient health data, elimination of paper, ink, and manual document printing, and reduction of phone calls and follow-ups with labs for test results. In addition, participation in HIEs is a requirement for HPs to receive stimulus funds from the government (Page 2010). It is possible to categorize the benefits as operational and quality benefits as well as the monthly portion of the long term funds and reimbursements awarded by the federal government and other agencies (Dixon et al. 2010). On the other hand, the costs include the average monthly cost of purchases and maintenance of equipment, IT infrastructure including the cost of interface construction, and personnel training.

Before joining the network, all HPs can estimate the net benefit using the methodologies mentioned in the HIE value analysis literature (e.g., Walker et al. 2005). If there are more participants
in the network, there is more information to share, and therefore the HIE membership becomes more valuable to its members. Hence, the benefit any HP expects from the collaboration increases in the network size, and is proportional to the number of participating HPs in the network (Calhoun 2013). Therefore, given that the average monthly benefit parameter for HP \( i \) is \( r_i \) and the network size is \( N_b \), the net benefit HP \( i \) would obtain from the collaboration is \( r_i N_b \). Our approach is in line with several models that consider the network externalities to be linear in the size of the consumer base (e.g., Fudenberg and Tirole 2000, Prasad et al. 2010).

Please note that the benefit parameters are not necessarily correlated with observables such as the size of HPs. For example, a large HP that already has a sophisticated IT system in place might find the benefit of HIE low compared to a small HP that has only basic IT functionalities. In addition, some HPs that have large amounts of patient data might be reluctant to share this data. Hence, they would find the HIE less beneficial. Moreover, in the general course of providing care, some HPs might already share some medical information among each other. However, since such transactions are generally not automated, these HPs should still benefit from the HIE membership. In general, because of such complications, it is usually not possible for the HIE provider to determine the amount of benefit each HP gains from joining the network. Therefore, we consider that the information about \( r_i \) is private in our model, i.e., it is unknown to the HIE provider and other HPs - but known to HP \( i \) only. However, the parties can estimate the “distribution” of \( r_i \) based on the available information regarding HPs.

As discussed earlier, each participating HP may request the value-added service in addition to the base HIE service. The HIE provider bundles the value-added services it offers, and we denote the level of this service by \( s \). Note that our analysis can be extended to analyze more than one service bundle, however that would not affect our key findings in this paper – it merely creates more cases to examine. Value-added services, such as e-prescribing, patient management tools, and quality reporting, have positive network affects. This implies that if more HPs in the network sign
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up for the value-added services, these services will be perceived as more valuable by HPs that have already signed up. In contrast to the general trend in network externality literature (Fudenberg and Tirole 2000, Jing 2007, Prasad et al. 2010, Niculescu et al. 2012), we do not assume that the benefit of value-added service is the same for all HPs. In some recent models, product valuations and the benefits customers obtain from the network are correlated (e.g., Sundararajan 2003, 2004). The latter approach is more appropriate in the case of HIEs. Hence, we consider that the valuations of both the base HIE service and the value-added service are different for different HPs.

Therefore, we formulate the expected benefit from the value-added service as $r_i s E[N_v] d$, where $r_i$ is the benefit parameter of HP $i$, $s$ is the level of the value-added service that is determined by the HIE provider, $N_v$ is the number of HPs that request the value-added service from the HIE provider, $E[N_v]$ is its expectation, and $d$ is a scaling parameter. Hence, if HP $i$ requests the value-added service, its expected net value from the HIE is $r_i s E[N_v] d + r_i E[N_b]$.

In this setting, an HP must be the part of the HIE network in order to request value-added service. As discussed before, there are two networks with separate but related externalities, and membership to one of them is a requirement for the membership to the other. Binary decision variables $q_{bi}$ and $q_{vi}$ equal to one if HP $i$ decides to join the HIE and request value-added service. If it does not join the HIE, both $q_{bi}$ and $q_{vi}$ are equal to zero. Otherwise, if it joins the HIE but does not request value-added service, then $q_{bi}$ equals to one and $q_{vi}$ equals to zero. Since value-added service requires base HIE subscription, we have $q_{bi} \geq q_{vi}$. This condition implies $N_b \geq N_v$, i.e., the number of HPs that request the value-added service cannot be more than the number of participating HPs in the network. Note that the value-added service may help increase the network size – in such a case, some HPs start participating in the network in order to benefit from value-added service. However, as we discuss in Lemma 5 in detail, value-added service does not always encourage new HPs to join the HIE network.
If HP $i$ does not request value-added service, but participate in the network, its objective function value is $r_i E[N_b] - p_b$, i.e., the difference between benefit and fee of the network subscription.\footnote{Hereafter, for simplicity, we refer to the objective function values of HP and HIE provider as simply their \textit{values}.} In order for HP $i$ to stay in the network, its benefit and cost difference needs to be positive (either when it merely joins the HIE, or when it joins the HIE and requests value-added service). The HIE provider collects fees from all HPs that join. Hence, the higher price it sets, the more revenue it collects per HP that are in the network. However, a high price for the HIE service deters the HPs because of two reasons: (i) direct effect of the higher fee, and (ii) the reduction in the benefit from the network externalities. In effect, a price increase implies fewer participants in the network, and in-turn even lower benefits to the remaining participants. Therefore, the HIE provider needs to balance the subscription price accordingly that determine the most beneficial network size. In addition, the HIE provider needs to determine: (i) the fee of value-added service (denoted by $p_v$) that is collected from all HPs that request the value-added service, (ii) and the level of value-added service (denoted by $s$) that is offered to the participants. The HIE provider also needs to take into account the effects of $p_v$ and $s$ on the network dynamics.

In addition to the subscription fees for the base HIE service and value-added service, the HIE provider receives funds from government and other third parties. A portion of this fund is directly proportional to the network size, whereas the other portion is fixed (independent of the network size) (Covich et al. 2011). We model the network dependent reimbursement as $N_b g$, where $g \geq 0$, and the fixed portion as $J$.

On the other hand, the costs for maintaining the HIE include implementation costs, hosting and data services costs, and administrative and operational costs (Covich et al. 2011). According to Mostashari et al. (2009), the cost for maintaining the network is positively correlated to the network size. Therefore, we model this cost as $c_b N_b + H_b$, where $c_b \geq 0$ and $g < c_b$. In this formulation, $H_b$ denotes the monthly adjusted fixed cost of establishing and maintaining the HIE network, and $c_b$
is the monthly cost of providing base HIE service to each HP in the HIE. In addition, the total cost of providing the value-added service increases with the total number of HPs that request it (i.e., \( N_v \)). Our discussions with the HIE providers reveal that they incorporate less costly, more valuable value-added services first in the bundle they offer. This implies that there is decreasing returns to scale with respect to the offered level of value-added service. Hence, we formulate the total cost of offering value-added service as: \( c_v N_v s^2 + H_v \).

Here, \( c_v \) is the cost multiplier term per HP that requests value-added services, and \( H_v \) is the periodic fixed cost of providing value-added service.

In summary, we can write the objective functions of the HIE provider and the HPs as:

\[
\begin{align*}
\max_{p, p_b, p_v} & \quad p_v \mathbb{E}[N_v] + p_b \mathbb{E}[N_b] + g \mathbb{E}[N_b] - c_b \mathbb{E}[N_b] - c_v \mathbb{E}[N_v] s^2 - H_b - H_v + J \\
\text{s.t.} & \quad q_{bi} \geq q_{vi} \quad \forall i \\
& \quad p_v \mathbb{E}[N_v] + p_b \mathbb{E}[N_b] + g \mathbb{E}[N_b] - c_b \mathbb{E}[N_b] - c_v \mathbb{E}[N_v] s^2 - H_b - H_v + J \geq 0 \\
& \quad q_{bi}, q_{vi} \in \{0, 1\} \quad \forall i.
\end{align*}
\]

In this formulation, Constraint (1) is the objective function of the HIE provider that tries to maximize its expected net value. Here, the decision variables of the HIE provider are the level of the value-added service, the fee for the value-added service, and the fee for the HIE subscription. Constraints (2) and (3) depict that each HP chooses among (i) staying out of the HIE network, (ii) joining the HIE, and (iii) joining the HIE and requesting value-added service. Moreover, due to Constraint (3), an HP cannot request value-added service if it does not join the HIE network.

Next, Constraint (4) ensures that the expected value of the HIE provider is non-negative. Finally,\(^5\) We can incorporate different cost structures in the model, rather than the quadratic cost form we employ. However, doing so would not change the insights of the paper as long as there is decreasing returns to scale with respect to the value-added service level.
Constraint (5) lists the binary decision variables. Note that parameters $H_b$, $H_v$, and $J$ are fixed amounts, and hence they do not have a significant effect on our analysis. Therefore, in order to keep the discussions and analysis concise, we set them to zero hereafter. However, key insights remain the same with non-zero levels of these parameters. In the next subsection, we present the benchmark case of our model.

3.2. Details of the Benchmark Case

In this section, we present the benchmark case of our model where the HIE provider does not offer HIE related value-added service. As discussed in Section 2.2, this setting represents most of the newly-established HIEs as they generally do not focus on value-added service. After removing the benefit and cost components related to the value-added services, we present the following benchmark case.

$$\max_{p_b} \quad p_b \mathbb{E}[N_b] + g \mathbb{E}[N_b] - c_b \mathbb{E}[N_b]$$

$$\max_{q_i} \quad q_i (r_i \mathbb{E}[N_i] - p_b) \quad \forall i$$

subject to

$$q_i \in \{0, 1\} \forall i$$

$$p_b \mathbb{E}[N_b] + g \mathbb{E}[N_b] - c_b \mathbb{E}[N_b] \geq 0.$$  

In this formulation, Equation (6) is the objective function of the HIE provider that tries to maximize its expected net value. Here, the decision variable of the HIE provider is the subscription fee (i.e., $p_b$). On the other hand, Equations (7) and (8) imply that the each HP joins the HIE only if its gain is positive. If it joins (i.e., $q_i = 1$), it receives value in the amount of $r_i \mathbb{E}[N_i] - p_b$. Otherwise, it stays out (i.e., $q_i = 0$) and obtains no value. Finally, Constraint (9) implies that the expected value of the HIE provider is non-negative. This constraint implies that in order for the HIE provider to establish the network, the business needs to be sustainable. Now, in the next subsection, we present our results and managerial insights.
4. Results and Managerial Insights

In this section, we discuss our key findings and present meaningful insights. In the following subsection, we present the results of the benchmark case of our model. After that, we provide the results of the main model.

4.1. The Benchmark Case

As discussed earlier, the HIE provider determines the price, and depending on this price, each HP decides whether to join or not. Further, the decision of other HPs (regarding whether they would join or not) affect the benefits of all participating members. Taking these into consideration, together with the fact that any HP would join the HIE only if it expects to have positive net value, we can write the following recursive relationship:

$$E[N_b|p_b] = N * P[r_i E[N_b|p_b] - p_b \geq 0].$$  

(10)

Without loss of generality, order \( r_i \) such that \( r_i < r_{i+1} \) \( \forall i \). Observe that if HP \( i \) joins the HIE, so does HP \( j \) where \( j > i \). In addition, if HP \( i \) is the HP with the lowest benefit parameter in the HIE, we can write \( E[N_b|p_b] = N \times (1 - F(r_i)) \), where \( F(.) \) is the cumulative distribution function of \( r_i \). In such a case, the HIE provider needs to set \( p_b \) to \( r_i E[N_b|p_b] =Nr_i (1 - F(r_i)) \). In other words, the HIE provider would not leave any value to the HP that is indifferent between joining the HIE or not. This is the price it would charge to all participating HPs. One interesting result is that the relationship between the price \( p_b \) and network size is not monotonic. This is because \( r_i (1 - F(r_i)) \) is not necessarily concave or convex in \( r_i \); in particular, the behavior depends on the distribution.

Next, denote the HP that is indifferent between joining and not joining at the equilibrium by \( i^* \), and the probability density function of \( r_i \) by \( f(.) \). Using the above arguments, the expression of the objective function for the HIE provider can be rewritten as:

$$N \times (1 - F(r_i)) \times (r_i \times N \times (1 - F(r_i)) - c_b + g).$$  

(11)
The first order condition with respect to $r_i$ reveals:

$$r_i^* = \frac{N(1 - F(r_i^*))^2 + f(r_i^*)(c_b - g)}{2Nf(r_i^*)(1 - F(r_i^*))}.$$  \hspace{1cm} (12)

In addition, the second order condition requires:

$$f'(r_i^*)(c_b + 2Nr_i^*F(r_i^*) - g - 2Nr_i^*) + 2Nf(r_i^*)f(r_i^*) + 2F(r_i^*) - 2 < 0.$$  

Hence, from this recursive relationship, we can derive the equilibrium network size. The above equality helps us glean some other insights, with some assumptions on the distribution of $r_i$. For example, if the distribution of $r_i$ has a decreasing failure rate (DFR), the equilibrium price “may” decrease with $c_b$. In such a case, the HIE provider should not decrease the network size by increasing the price, even though increasing the price would make the HIE provider collect more value from each of the remaining participants. Instead, the HIE provider should increase the network size by decreasing the price, and collect more value in total from all of the participants in the HIE.

Next, we would like to answer the following questions using the results of our model: (i) how does variation in the distribution affect participation in the HIE network, and (ii) how does the behavior of net values of the parties change with respect to different characteristics of HPs and HIE provider? In order to explore the answers to these questions and several others, more structure in the distribution of $r_i$ is needed. All of our key insights hold with any distribution that is bounded from above and below and is unimodal. Further, most of our insights hold with any distribution that is unimodal (but not necessarily bounded). However, in order to be conservative about the variance of the distribution, we consider that $r_i$ follows a uniform distribution with lower and upper bounds $r_-$ and $\bar{r}$, i.e., $r_i \sim U(r_-, \bar{r})$. For the rest of this paper, the heterogeneity refers to the variability in benefit parameters of HPs. In particular, we measure the heterogeneity with the range of the distribution, i.e., $\bar{r} - r_-.$

The solution of the game among the parties reveals the equilibrium network size that is presented in Lemma 1 below.
Lemma 1. The equilibrium network size is:

(a) \( \frac{N^{r+1} \sqrt{N(Nr^2 - 3(r-\bar{c})(c_b - g))}}{3(r-\bar{c})} \) if \( c_b < g + \frac{r^2 N}{4(r-\bar{c})} \) and \( r > 3r \), or \( g + N(3r - \bar{r}) \leq c_b \leq g + \frac{r^2 N}{4(r-\bar{c})} \) and \( 2r = \bar{r} \leq 3r \).

(b) \( N \) if \( c_b \leq g + \frac{r^2 N}{4(r-\bar{c})} \) and \( \bar{r} < 2r \), or \( c_b < g + N(3r - \bar{r}) \) and \( 2r \leq \bar{r} \leq 3r \).

(c) 0 if \( c_b > g + \frac{r^2 N}{4(r-\bar{c})} \).

If heterogeneity is large, or if both heterogeneity and the cost of keeping each HP in the HIE are at medium levels, then a partial network is established where some HPs join but not all (as presented in part (a) of Lemma 1). The second and third cases, on the other hand, depict the cases when every HP joins the HIE and when the HIE is not established, respectively. The key factor that determines whether or not the HIE is established is the cost for hosting each HP in the network. If \( c_b \) is more than the threshold \( g + \frac{r^2 N}{4(r-\bar{c})} \), the HIE provider cannot sustain an HIE and it does not establish one (as shown in part (c) of Lemma 1). This threshold value decreases with an increase in the maximum benefit HPs can get from the HIE (i.e., \( \bar{r} \)) when the heterogeneity among the HPs is small (in particular, if \( \bar{r} < 2r \)). Hence, if the heterogeneity is small, contrary to intuition, when maximum benefit HPs can get from the HIE increases, the cost of accommodating each HP in the network has to be even lower in order to establish the HIE.

It is interesting to note that, if the above condition is satisfied (i.e., \( \bar{r} < 2r \)) and the cost is less than the above mentioned threshold value, then all of the HPs decide to join the HIE network (as shown in part (b) of the lemma). If the heterogeneity is relatively higher (i.e., \( 2r \leq \bar{r} \leq 3r \)), the cost needs to be even less than \( g + N(3r - \bar{r}) \) for full participation. If the heterogeneity is even higher (\( \bar{r} > 3r \)), full participation is not possible if government and third party incentives do not cover the entire subscription fee (i.e., \( g \neq c_b \)). In such a case, the HIE provider finds it more beneficial to increase the subscription fee and collect more value from the participating HPs, rather than setting a low price and enticing all HPs to join the network. We summarize this finding in the following proposition.
PROPOSITION 1. In general, heterogeneity among HPs decrease the number of HPs in the network. Also, the full participation is not possible if the heterogeneity among the HPs is high, i.e., $r < \frac{r_3}{r}$.

Hence, when the heterogeneity is high, or when costs related to providing HIE services are high, policy-makers and HIE providers may employ the following strategies to encourage participation. First, the government or HIE providers can focus on providing more incentives to HPs so that the minimum benefit HPs can get from the participation is increased. For example, if small HPs do not have functional EHR implementations, HIE providers might offer EHR applications for free or at a very low price. In the U.S., several HIE providers are already offering this incentive. However, as apparent in Proposition 1, these incentives cannot entice all HPs to join the network when the heterogeneity among HPs is high. Hence, as a second option, more than one HIE can be established in the region. Establishing a second HIE in the same region that only focuses on low-gain HPs might increase the benefits of low-gain HPs, and in turn, more of them might decide to join the HIE. Hence, under certain conditions, there might be an incentive for the policy-makers to establish more than one HIE in a region. The third option to entice more HPs is to provide HIE related value-added service that we analyze in the next subsection.

Our analysis and the results in Lemma 1 also reveal that if the HIE network is established, at least half of the HPs in the region find it beneficial to join the network. We summarize this finding in the following proposition.

PROPOSITION 2. If the HIE is established, at least half of the HPs in the region join the network at the equilibrium.

Having multiple HIE providers may not be economically feasible, if, for example, the fixed cost of maintaining an HIE is prohibitively large. Even if there is a second HIE provider, it might not be able to induce full participation, but it can at least increase participation. We skip the formal analysis where there are more than two HIEs in the region, because such analysis would not add other (foreseeable) valuable insights.
As discussed earlier, the number of participating HPs affects the benefits of the HPs that are in the network. Proposition 2 reveals that if any party conjectures that the HIE would not be beneficial to more than half of the HPs in the region, then the HIE would not be established. Therefore, the HIE provider and the policy-makers should consider this when they are designing incentive schemes that affect how much the HIEs would be beneficial to the participating HPs.

If the government changes the amount of incentives for HPs, these are reflected in changes in the upper and lower bounds of the distribution of the benefit parameter (hence, in heterogeneity as well). Therefore, in the following proposition, we examine how network size is affected if the benefit parameters change. We focus on the partial network case presented in part (a) of Lemma 1 since other cases are trivial.

Proposition 3. When there is a partial network, its size:

(a) Increases with the lowest benefit HPs can get from the HIE \( (r_l) \).

(b) Increases with the highest benefit HPs can get from the HIE \( (r_u) \) if and only if the heterogeneity and the cost of accommodating each HP in the HIE are both moderately high, in particular, if \( r > 4r \) and \( c_0 > g + \frac{4rN}{3} \).

A higher lower bound \( r \) (while keeping the upper bound \( r_u \) constant) implies that the heterogeneity decreases. In such a case, as stated in the first part of Proposition 3, participation in the HIE increases. On the other hand, as presented in the second part of the proposition, under certain conditions, the equilibrium network size increases with an increase in heterogeneity. This happens when both heterogeneity and the cost are moderately high. In such a case, the HIE provider finds it more beneficial to entice more HPs into the network. On the other hand, if the heterogeneity is not large, in particular if \( r < 4r_u \), introducing more benefits to the HPs by increasing the upper bound actually decreases the participation in the network. In those cases, the HIE provider focuses
on high-gain HPs in order to collect more value from them. Since the goal of the policy-makers is to increase participation in HIEs, they should be careful when designing incentive schemes for HPs.

Next, we present the equilibrium subscription fee that is collected from each of the participating HP for the case when there is partial participation in the HIE. Equilibrium subscription fees for full participation and no participation cases are trivial, and hence not reported. The subscription fee presented in Lemma 2 enables the HIE provider to collect the most value from the network given that it enables the provider to extract all value from the HP that is indifferent between joining and not joining the HIE.

**Lemma 2.** When there is partial participation, the equilibrium subscription fee that is collected from each HP in the HIE is given by:

\[ p^*_b = \frac{N\bar{r}^2 + 3(\bar{r} - \bar{r})(c_b - g) + \bar{r}\sqrt{N\left(N\bar{r}^2 - 3(\bar{r} - \bar{r})(c_b - g)\right)}}{9(\bar{r} - \bar{r})^2}. \]

Clearly, the expected network size decreases in network subscription price \( p_b \). In addition, sensitivity analysis of \( p^*_b \) given in Lemma 2 reveals that it increases in \( \bar{r} \) and \( \bar{r} \). This finding seems reasonable because, as the participants expect more benefits, the HIE provider extracts more value from them by increasing the price. Now, based on Lemmas 1 and 2, we present and discuss the objective function values of the HPs and the HIE provider. Again, we present the results only for the case when a partial network is formed.

**Lemma 3.** When a partial network is formed, the values different HPs obtain from the HIE and the value of the HIE provider are as follows:

(a) The value of HP \( i \) is

\[ q_i \left( \frac{N(3\bar{r}_i - 2\bar{r}) + \sqrt{N(N\bar{r}^2 - 3(\bar{r} - \bar{r})(c_b - g))}}{9N(\bar{r} - \bar{r})}\right) \left( N\bar{r} + \sqrt{N(N\bar{r}^2 - 3(\bar{r} - \bar{r})(c_b - g))} \right). \]

(b) The value of the HIE provider is

\[ \frac{\left(N\bar{r}^2 - 6(\bar{r} - \bar{r})(c_b - g) + \bar{r}\sqrt{N(N\bar{r}^2 - 3(\bar{r} - \bar{r})(c_b - g))}\right)}{27(\bar{r} - \bar{r})^2} \left( N\bar{r} + \sqrt{N(N\bar{r}^2 - 3(\bar{r} - \bar{r})(c_b - g))} \right). \]
The sensitivity analysis of the equilibrium values presented in Lemma 3 with respect to different parameters reveal several interesting results. First, for all participating HPs, the net value they receive from the HIE decrease if there is an increase in the highest benefit an HP can get from the network (i.e., $r$) although such an increase seems beneficial to the participating HPs. In such a case, the HIE provider extracts more value from the participating HPs by increasing the price. Hence, the policy-makers or the HIE provider should proactively seek ways to propose more value to all participants (by increasing $r$) when $r$ increases. However, this should be done with a grain of salt. This is because, if the benefit parameter of an HP is less than $\frac{r}{3}$, an increase in $r$ actually decreases its net value. This result might seem counter-intuitive, but can be explained as follows. Increasing $r$ increases both the equilibrium subscription price (see the discussion after Lemma 2) and the equilibrium network size (see part (a) of Proposition 3). Therefore, HPs whose benefits are more sensitive to the network size (i.e., $r_i > \frac{r}{3}$) realize a net benefit if $r$ increases in spite of the price increase. Rest of the HPs do not find it beneficial because the extra benefit they get from the larger network cannot compensate the increase in the subscription fee. We summarize this discussion in the following proposition.

**Proposition 4.** The value of HP $i$ at the equilibrium:

(a) Decreases with the highest benefit HPs can get from joining the HIE.

(b) Increases with the lowest benefit HPs can get from joining the HIE if and only if $r_i > \frac{r}{3}$.

The sensitivity analysis of the equilibrium value of the HIE provider (presented in Lemma 3) reveals that it increases in $r$. On the other hand, we know from part (a) of Proposition 4 that the values of HPs actually decrease in $r$. Hence, an increase in heterogeneity because of an increase in the upper bound is beneficial only to the HIE provider. We now analyze the impact of decrease in heterogeneity with an increase in the lower bound $r$. The sensitivity of the equilibrium value of the HIE provider with respect to $r$ shows that HIE provider’s value increases with $r$. However,
as discussed in part (b) of Proposition 4, an increase in $r$ increases the value of HPs only if they are high-gain HPs, i.e., if $r_i > \frac{\gamma}{3}$. Hence, decreasing the heterogeneity by increasing $r$ is also not beneficial to all parties.

The results presented in this section not only serve as a benchmark to our main model, but also provide several useful guidelines for policy-makers and many start-up HIEs, such as SETHS, with regard to setting the subscription fee in those environments where the HIE provider offers only the base HIE service. Now, in the next subsection, we discuss our findings in the main model where the HIE provider also offers HIE related value-added service to the participating HPs.

### 4.2. Results and Managerial Insights of the Main Model

In the main problem, as discussed earlier, the HIE provider sets: (i) the price of base services (i.e., $p_b$), (ii) the price of value-added services (i.e., $p_v$), and (iii) the level of the value-added services. Depending on these, each HP decides whether to join the network or not, and whether to request value-added services or not. As discussed before, the decisions of other HPs regarding whether they would join the HIE (resp., request value-added services) affect the benefits of all participating HIEs from joining the HIE (resp., from value-added services). There are two possible cases: (i) some of the participating HPs do not request value-added services, i.e., $\mathbb{E}[N_v] < \mathbb{E}[N_b]$, and (ii) each participating HP requests value-added services, i.e., $\mathbb{E}[N_v] = \mathbb{E}[N_b]$. For $\mathbb{E}[N_v] < \mathbb{E}[N_b]$, we can write the following recursive relationships:

$$
\mathbb{E}[N_b|p_b] = N \ast \mathbb{P}[r_i \mathbb{E}[N_b|p_b] - p_b \geq 0],
$$

$$
\mathbb{E}[N_v|p_v] = N \ast \mathbb{P}[r_i \mathbb{E}[N_v|p_v]d - p_v \geq 0].
$$

On the other hand, if $\mathbb{E}[N_v] = \mathbb{E}[N_b]$, we have the following relationship:

$$
\mathbb{E}[N_b|p_b, p_v] = N \ast \mathbb{P}[r_i \mathbb{E}[N_b|p_b, p_v]d + r_i \mathbb{E}[N_b|p_b, p_v] - p_v - p_b \geq 0].
$$

Without loss of generality, order $r_i$ such that $r_i < r_{i+1} \ \forall i$. Observe that if HP $i$ joins the HIE or requests value-added services, so does HP $j$ where $j > i$. Therefore, if $\mathbb{E}[N_v] < \mathbb{E}[N_b]$, and if HP $i_b$ is
the HP with the lowest benefit parameter that joins the HIE, we have $\mathbb{E}[N_b | p_b] = N \left(1 - F \left( r_b \right) \right)$. Besides, if HP $i_v$ is the HP with the lowest benefit parameter in the HIE that requests value-added services, we have $\mathbb{E}[N_v | p_v] = N \left(1 - F \left( r_v \right) \right)$. Here, $F(.)$ is the cumulative distribution function of $r_i$. In such a case, the HIE provider needs to set $p_b = r_b \mathbb{E}[N_b | p_b] = N r_b \left(1 - F \left( r_b \right) \right)$, and $p_v = r_v s \mathbb{E}[N_v | p_v] d = r_v s N \left(1 - F \left( r_v \right) \right) d$. These equalities imply that the HIE provider does not leave any value to the HP that is indifferent between joining and not joining, and indifferent between requesting value-added services or not.

It is possible to glean some insights from our model even without making assumptions on the distribution of the benefit parameter $r_i$. However, in order to explore our model to a greater extent and to get more results, more structure in the distribution of $r_i$ is needed. In order to be conservative about the variance, we consider that $r_i$ follows a uniform distribution with lower and upper bounds $\underline{r}$ and $\bar{r}$, i.e., $r_i \sim \mathcal{U} (\underline{r}, \bar{r})$.

Denote the HP that is indifferent between joining and not joining at the equilibrium by $i^*_b$, its benefit parameter by $r^*_b$, the equilibrium subscription price set by the HIE provider by $p^*_b$, and the equilibrium network size by $N^*_b$. In addition, denote the HP that is indifferent between requesting value-added services or not at the equilibrium by $i^*_v$, its benefit parameter by $r^*_v$, the equilibrium price of the value-added service by $p^*_v$, the optimal level of value-added service by $s^*$, and the number of HPs that request value-added services at the equilibrium by $N^*_v$. Hence, it is easy to show that $\mathbb{E}[N_b | p^*_b] = N \frac{r^*_b - \underline{r}}{\bar{r} - \underline{r}}$, and $\mathbb{E}[N_v | p^*_v] = N \frac{r^*_v - \underline{r}}{\bar{r} - \underline{r}}$. Therefore, after substituting these into the objective functions of the HIE provider and the HPs, we can solve the game and derive equilibrium values. In the next lemma, we present the equilibrium levels of the value-added service, and the number of HPs that request value-added services.

**Lemma 4.** When the HIE provider offers value-added services, the equilibrium level of the value-added service (i.e., $s$), and the expected number of HPs that request value-added services given the
fee structure (i.e., $E[N_v|p^*_v]$) are given by:

$$s = \frac{N r^*_v (r^*_v - r^*_u) d}{2 c_v (r^*_v - r^*_u)},$$

$$E[N_v|p^*_v] = N \frac{r^*_v - r^*_u}{r^*_v - r^*_u},$$

given that the following holds:

$$E[N_v|p^*_v] \leq E[N_b|p^*_b].$$

Lemma 4 implies that there is a recursive relationship between the number of HPs that request the value-added service and the level of the value-added service. Taking this into account, the HIE provider can optimize the level of value-added service it offers to HPs, the fee of the value-added service, and the fee for participation in the HIE. Below, we present the equilibrium number of participants in the HIE, the equilibrium number of HPs that are interested in value-added services, and the conditions that enable the HIE provider to increase the number of participants in the network because of the value-added services. We relegate our discussion pertaining to the optimal fee structure to Lemma 6. For conciseness, we use the threshold value $\Gamma$ to represent

\[
\frac{729 r^5 N^3 d^2}{3125 (r^* - \mu_c) (r^* - \mu_b)} \left( \frac{3 (r_b - g) (r^* - \mu_c)}{2 \sqrt{N (r^* N - 3 (r_b - g) (r^* - \mu_c)) + 3 \pi N}} \right)^2 - 2 r^2 N \left( \sqrt{N (r^* N - 3 (r_b - g) (r^* - \mu_c)) + \pi N} \right)
\]

in part (a) of Lemma 5.

**Lemma 5.** When the HIE provider optimizes the level of value-added services, the equilibrium network size (i.e., $N_b$), and the number of HPs that request value-added services (i.e., $N_v$) are given by:

(a) If the cost of maintaining each HP in the HIE is high, but the cost of providing value-added services for each HP is low, then the HIE is established because of an interest in the value-added services. On the other hand, if both of these costs are high, the HIE is not established, and hence no HP can request value-added services. In particular:

1. If $c_b > g + \frac{r^5 N}{4 (r^* - \mu_c)}$ and $c_v < \Gamma$, then $0 \leq N_b = N_v \leq \min \left\{ \frac{3 \pi N}{4 (r^* - \mu_c)}, N \right\}$. 

(2) If \( c_b > g + \frac{r^2 N}{3(r - 2)} \) and \( c_v > \Gamma \), then \( N_b = N_v = 0 \).

(b) If the cost of maintaining each HP in the HIE is relatively lower, value-added services help increase the network size in two settings. In particular, \( \frac{N \tau + \sqrt{N(N^2 - 3(r - 2)(c_b - g))}}{3(r - 2)} < N_b = N_v \leq \min \left\{ \frac{3\tau N}{5(r - 2)}, N \right\} \), if

\[
\begin{align*}
(1) & \quad g + \frac{r^2 N}{25(r - 2)} < c_b < g + \frac{r^2 N}{4(r - 2)} \quad \text{and} \quad \tau > 3\tau_v, \quad \text{or} \\
(2) & \quad \max \left\{ (g + N(3\tau_v - \tau)), \left( g + \frac{r^2 N}{25(r - 2)} \right) \right\} \leq c_b \leq g + \frac{r^2 N}{4(r - 2)} \quad \text{and} \quad 2\tau_v \leq \tau \leq 3\tau_v .
\end{align*}
\]

(c) If the cost of maintaining each HP in the HIE is low so that all HPs are already in the HIE, some portion or all of the HPs might request the value-added services under two circumstances.

In particular, \( N_b = N \geq N_v = \min \left\{ \frac{3\tau N}{5(r - 2)}, N \right\} \), if

\[
\begin{align*}
(1) & \quad c_b \leq g + \frac{r^2 N}{4(r - 2)} \quad \text{and} \quad \tau < 2\tau_v, \quad \text{or} \\
(2) & \quad c_b < g + N(3\tau_v - \tau) \quad \text{and} \quad 2\tau_v \leq \tau \leq 3\tau_v .
\end{align*}
\]

(d) If there were a partial network without value-added services, and if the cost is relatively small, only a portion of HPs in the HIE would request value-added services. This is true under two circumstances. In particular, \( N \geq N_b = \frac{N \tau + \sqrt{N(N^2 - 3(r - 2)(c_b - g))}}{3(r - 2)} \geq N_v = \frac{3\tau N}{5(r - 2)} \), if

\[
\begin{align*}
(1) & \quad c_b < g + \frac{r^2 N}{25(r - 2)} \quad \text{and} \quad \tau > 3\tau_v, \quad \text{or} \\
(2) & \quad g + N(3\tau_v - \tau) \leq c_b \leq g + \frac{r^2 N}{4(r - 2)} \quad \text{and} \quad c_b \leq g + \frac{r^2 N}{25(r - 2)} \quad \text{and} \quad 2\tau_v \leq \tau \leq 3\tau_v .
\end{align*}
\]

If the HIE provider does not offer value-added services, the HIE network cannot be established when the cost of accommodating each HP in the HIE (i.e., \( c_b \)) is more than \( g + \frac{r^2 N}{4(r - 2)} \) (as discussed in part (c) of Lemma 1). On the other hand, in such a case, if the value-added service is provided and its cost per HP is sufficiently low (i.e., \( c_v < \Gamma \)), the HIE is established and each of the participating HPs request the value-added service (as shown in part (a1) of Lemma 5). In this case, these HPs join the network mainly because of the value-added service. In some other cases outlined in part (b) of Lemma 5, the value-added service helps the HIE provider enlarge the network. On the other hand, in parts (c) and (d) of Lemma 5, only a portion of the HPs that are already in the network request
the value-added service. Therefore, in these cases, the HIE network does not expand because of
the value-added service.

Heterogeneity among the HPs does not play a significant role in determining whether or not value-added services would increase participation in the HIE. However, it is apparent from parts (a) and (b) of Lemma 5 that the cost of accommodating each HP in the HIE (i.e., $c_b$) plays an important role. In addition, although the cost of value-added service per HP (i.e., $c_v$) does not appear in part (b) of the lemma, it affects the participation in the cases depicted in part (a). Hence, the key take-away is that, if the cost of accommodating each HP in the HIE is prohibitively large, the government might consider reimbursing costs related to providing value-added services. That would reduce $c_v$ and may promote participation in the network. On the other hand, if the cost of accommodating each HP in the HIE is not prohibitively large, reimbursing costs related to providing value-added services does not entice more participation in the network.

Now, in the following lemma, we present the optimal levels of the value-added service and the fees. For brevity, we present and analyze the results only for part (d) of Lemma 5. This case possibly represents one of the most interesting scenarios where (i) the partial network existed even without the value-added services, and (ii) the network size does not increase because of the value-added services.

**Lemma 6.** For part (d) of Lemma 5, the optimal level of value-added service (i.e., $s^*$), optimal value-added service fee (i.e., $p_v^*$), and the optimal HIE network subscription fee (i.e., $p_b^*$) are given by:

\[
s^* = \frac{3\bar{r}^2 N d}{25c_v(\bar{r} - \underline{r})},
\]

\[
p_v^* = \frac{18\bar{r}^4 N^2 d^2}{625c_v(\bar{r} - \underline{r})^2},
\]

\[
p_b^* = \frac{N\bar{r}^2 + 3(\bar{r} - \underline{r})(c_b - g) + \bar{r}\sqrt{N\left(N\bar{r}^2 - 3(\bar{r} - \underline{r})(c_b - g)\right)}}{9(\bar{r} - \underline{r})}.
\]
The sensitivity analyses of these optimal values with respect to benefit parameters $\tau$ and $\underline{\tau}$ yield the following insights. First, all of these values increase with benefit parameters. Hence, an important question arises: When HPs expect more benefits from the HIE, would there be (i) more HPs interested in value-added services (because of the increased level of the service), or (ii) less HPs interested in value-added services (because of the increase in fees for both the value-added service and the base HIE subscription)? Interestingly, part (d) of Lemma 5 reveals the following result.

**Proposition 5.** For part (d) of Lemma 5:

(a) If the maximum benefit (i.e., $\tau$) is increased, the network size would decrease and a smaller number of HPs would request the value-added service.

(b) If the minimum benefit (i.e., $\underline{\tau}$) increases, more HPs would join the HIE and more HPs would request the value-added service.

This result can be explained as follows. When the maximum benefit increases, the HIE provider is able to collect more value from a smaller subscriber base. On the other hand, when the minimum benefit parameter increases, it is beneficial for the HIE provider to expand its network and provide value-added services to a larger group of HPs. Now, in the following lemma, we present the equilibrium net values of HPs and the HIE provider. Again, we focus on the conditions presented in part (d) of Lemma 5.

**Lemma 7.** For part (d) of Lemma 5, the net values of HP $i$ and the HIE provider at the equilibrium are given by:

$$\frac{81\tau_i^3 N^2(5\tau_i - 2\tau)^2 + 625c_v(\tau - \underline{\tau})\left(N^2 - 3(c_b - g)(\tau - \underline{\tau})\right)}{5625c_v(\tau - \underline{\tau})^2},$$

and

$$\frac{72\tau^5 N^3 d^2 + 3125c_v(\tau - \underline{\tau})\left(N^2 - 6(c_b - g)(\tau - \underline{\tau}) - 3(c_b - g)N(\tau - \underline{\tau})\right)}{84375c_v(\tau - \underline{\tau})^3},$$

respectively.

Sensitivity analyses of these values with respect to different parameters reveal several interesting results. First, we find that although the heterogeneity among HPs plays an important role in
determining the behavior of values of all participating HPs, the individual benefit parameters of HPs (i.e., $r_i$) have the most vital role. In particular, a change in any parameter might imply opposite effects for high gain (HPs that have high $r_i$ values) and low gain HPs (HPs that have low $r_i$ values). We present one such finding in the following proposition, along with some other results.

**Proposition 6.**

(a) If the highest benefit the HPs can get from the HIE (i.e., $\tau$) increases, the value of HP $i$ at the equilibrium

1. Decreases if the benefit parameter $r_i$ is less than $\frac{4}{5}\tau$.
2. Might increase if the benefit parameter $r_i$ is more than $\frac{4}{5}\tau$, the heterogeneity is high, and the value-added service is more beneficial (i.e., $d$ is high).

(b) If the minimum benefit HPs can get from the HIE (i.e., $\underline{\tau}$) increases, the equilibrium values of all HPs increase.

When the HIE provider does not offer value-added services, an increase in the maximum benefit HPs can get from the HIE decreases the values of all HPs (as explained in part (a) of Proposition 4). However, when the HIE provider offers the value-added service, an increase in the highest benefit the HPs can get from the HIE actually increases the value of participants in some cases (as outlined in Proposition 6(a)). Hence, value-added services can actually help HPs in extracting more value from their HIE membership even when the heterogeneity among them increases because of an increase in $\tau$.

Proposition 6(b) shows that the equilibrium values of all HPs increase if the minimum benefit HPs can get from the HIE (i.e., $\underline{\tau}$) increases. On the other hand, when the HIE provider does not offer the value-added service, an increase in $\tau$ does not necessarily increase the values of the participating HPs (as described in part (b) of Proposition 4). Hence, value-added services are
beneficial again because they ensure an increase in values of all participating HPs with an increase in $r$.

Finally, the HIE provider benefits from an increase in both $\bar{r}$ and $r$. This result is same as that in the benchmark case analyzed in the previous subsection. Now, in the next section, we discuss the implications of our research for different parties and propose future research topics.

5. Implications and Conclusions

This study is among the first of its kind to model the sustainability of healthcare information exchanges (HIEs). Our goal is to provide useful insights for all of the key parties involved in the sustainability of HIEs: (i) the healthcare practitioners (HPs), (ii) the HIE providers, and (iii) the policy-makers. In order to develop our main model, we worked with ICC and Critical Connection — two well-established HIEs based in Texas that offer value-added services to the participating HPs in addition to the base HIE service. The benchmark case of our model also helps us understand the issues involved in settings where the HIE provider does not offer value-added services. While building this model, we interacted with a start-up HIE SETHS that is based in Texas. Although our model and its benchmark case are based on these three HIEs, they are generic, and the results as well as insights are applicable to most of the HIE settings. In the next subsection, we discuss our key findings.

5.1. Implications

In the benchmark case of our model, we analyze a setting where the HIE provider offers only the base service (typical for many start-up HIEs). As expected, we find that the cost of accommodating each HP in the HIE has the most important role in determining whether an HIE is established or not. We find that if this cost is not prohibitive or if there is enough government support, the heterogeneity among HPs (in terms of their expected benefit from the HIE membership) determines the extent of participation in the HIE. In particular, we find that some low-gain HPs choose not to join the HIE if there are high-gain HPs in the region. We propose three different strategies
for policy-makers and HIE providers that can help boost participation in HIEs: (i) increase the minimum benefit HPs can get from the HIE, (ii) establish additional HIEs in order to focus on low-gain HPs, and (iii) incentivize and/or offer HIE related value-added services.

Increasing the minimum benefit the HPs can obtain from the HIE entices small HPs to join the network. The policy-makers can do so by increasing the amount of proposed funds and incentives. Another possible strategy is to offer electronic health record (EHR) applications for free or at a very low price to the HPs that do not have functional EHR. In the U.S., several HIE providers are already offering this incentive. On the other hand, increasing the maximum benefit the HPs can obtain from the HIE might even be detrimental to participation under certain circumstances. The reason is that increasing the minimum gain decreases the heterogeneity among HPs, whereas an increase in the maximum benefit increases the heterogeneity.

Besides, we find that although the incentives by the government and other agencies encourage a larger HIE network, these incentives cannot entice all HPs to join the network when the heterogeneity among the HPs is high. Hence, the policy-makers should be careful about when, to whom, and how much incentives to offer. The reason is that, in some circumstances, the HIE provider may try to extract more value if the values of participating HPs are increased. It may simply prefer a smaller HP pool that can afford a higher subscription fee, instead of a higher number of HPs with a lower subscription fee. We also find that, in the benchmark case, the HIE provider decides to establish the HIE network only if at least half of the HPs in the region find it beneficial to participate. Hence, the HIE provider should be careful in planning its strategies regarding which HPs to focus.

The second option is that, an additional HIE can be established that should focus on developing solutions for low-gain HPs and increasing their benefits. This would entice more HPs to join the network. Hence, under certain conditions, there is an incentive for the policy-makers to establish
more than one HIE in the same region. SETHS has a focus on rural hospitals, and this seems to
be an effective practice that would increase participation in its region.

As a third option, HIE providers can offer HIE related value-added services that might also help
increase participation in their networks. Therefore, in our main model, we examine how the HIE
provider decides on the level of the value-added service and its fee, and which HPs request these
services. In addition, we analyze the conditions when the value-added services expand the network.
We find that, in some cases, value-added services help sustain the HIE. In some of these cases, the
HIE is established only when the value-added services are offered to HPs. In some other cases, the
value-added services can increase the number of participants in the HIE. Therefore, policy-makers
should consider incentivizing HIE-related value-added services in order to increase the number of
participants in the HIE. In addition, we glean several other managerial insights in both settings
that would be useful for the policy-makers, the HIE providers, and the HPs.

5.2. Future Research Directions

There are several future research possibilities in the domain of sustainability of HIEs. Here, we
would like to discuss some of them. We already validated our modeling choices with the HIEs we
work with. However, it would be valuable to empirically validate our results using transaction and
encounter data from HIE providers. Here, the transaction data refers to the data regarding the
interactions of an HIE provider with the HPs, and the encounter data refers to the interactions of
HPs with the patients.

Some HIE providers are trying newer business models that are worth exploring. One such example
is the co-op services where an HP in the network is bound to make purchases of office or medical
supplies from third-parties (such as Staples and Office Max) that are selected by the HIE provider.
In these settings, the HIE provider gets a percentage of the revenues from such transactions. Hence,
there are three different parties involved in such a setting. In order to analyze this setting, one
needs to modify our model to include third-parties.
References


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Online Supplement to
Sustainability of Healthcare Information Exchanges: A Game-Theoretic Approach

Proofs of Lemmas 1, 2, and 3

Because $r_i$ follows the uniform distribution with parameters $\underline{r}$ and $\bar{r}$, we can replace $f(r_i^*)$ and $F(r_i^*)$ with $\frac{1}{\bar{r}-\underline{r}}$ and $\frac{r_i^* - \underline{r}}{\bar{r} - \underline{r}}$ respectively in Equation 12. Hence, we get a closed form expression for $r_i^*$ that we substitute into $E[N^*|p^*] = N(1 - F(r_i^*))$. This procedure reveals that when there is a partial network, its size is $\frac{N_{r_i^*}^* + \sqrt{N(Nr_i^* - 3(\bar{r} - \underline{r})(c_b - g))}}{3(\bar{r} - \underline{r})}$ as discussed in part (a) of Lemma 1.

The fact that the HIE provider would not leave any value to the HP that is indifferent between joining and not joining implies that the optimal price is set such that the following satisfies: $p^*_b = r_i^*E[N^*|p^*_b] = Nr_i^*(1 - F(r_i^*))$. Hence, after substituting $r_i^*$ and $F(r_i^*)$ into this equality, we get $p^*_b = \frac{Nr_i^* + \sqrt{N(Nr_i^* - 3(\bar{r} - \underline{r})(c_b - g))}}{3(\bar{r} - \underline{r})}$ as presented in Lemma 2.

Equilibrium values of the HPs and the HIE provider depicted in Lemma 3 are derived by substituting the solution into the objective functions of the parties presented in Equations 6 and 7.

Finally we would like to discuss parts (b) and (c) of Lemma 1. The derivation of the conditions presented in these parts of the Lemma is due to three constraints we have in our model. The first two constraints are related to the bounds on the network size. It is bounded below by 0 (i.e., no HP joins the network), and above by $N$ (i.e., every HP joins the HIE). Hence, we have: $0 \leq \frac{Nr_i^* + \sqrt{N(Nr_i^* - 3(\bar{r} - \underline{r})(c_b - g))}}{3(\bar{r} - \underline{r})} \leq N$. Equation 9 constitutes the basis for the third constraint. It implies that the HIE provider would not establish the HIE, if its expected value provided in Lemma 3 is below zero. Hence, consideration of these three constraints simultaneously, results in the conditions outlined in Lemma 1. Please note that second order conditions confirm our results.
Proof of Proposition 1

The presented condition is derived by examining when the partial network size can be made equal to \( N \). Part (b) of Lemma 1 reveals that it is possible when \( c_b < g + \frac{\pi^2 N}{4(r - \tau)} \) and \( \tau < 2r \), or \( c_b < g + N(3\tau - \tau) \) and \( 2\tau \leq \tau \leq 3\tau \). Hence, when \( \tau < \frac{\pi}{3} \), no matter what the values of cost (i.e., \( c_b \)) and reimbursement (i.e., \( g \)) parameters are (as long as the government does not cover all the costs, i.e., \( g < c_b \)), network size cannot be equal to \( N \).

Proof of Proposition 2

Observe that partial network size \( N_b = \frac{N\tau + \sqrt{N(N\tau^2 - 3(\tau - \tau)(c_b - g))}}{3(\tau - \tau)} \) strictly decreases in cost parameter \( c_b \). Part (a) of Lemma 1 states that \( c_b < g + \frac{\pi^2 N}{4(r - \tau)} \). Hence, if we substitute \( c_b = g + \frac{\pi^2 N}{4(r - \tau)} \) into \( N_b = \frac{\pi N}{2(\tau - \tau)} \), we get \( N_b = \frac{\pi N}{2(\tau - \tau)} \). Setting \( \tau \) to 0 results in the minimum possible network size that is \( \frac{N}{2} \).

Proof of Proposition 3

Proof for part (a) of the proposition is trivial, hence is omitted. Derivative of the equilibrium network size with respect to the upper bound parameter equals to: \( \frac{d(N_b)}{d\tau} = -\frac{N\left(2\tau\sqrt{N(N\tau^2 - 3(\tau - \tau)(c_b - g)) + 2\tau^2 N - 3(\tau - \tau)(c_b - g)}\right)}{6(\tau - \tau)^2\sqrt{N(N\tau^2 - 3(\tau - \tau)(c_b - g))}} \). We know from Lemma 1 that \( c_b < g + \frac{\pi^2 N}{4(r - \tau)} \). Hence, the derivative is positive if and only if \( N\left(2\tau\sqrt{N(N\tau^2 - 3(\tau - \tau)(c_b - g)) + 2\tau^2 N - 3(\tau - \tau)(c_b - g)}\right) < 0 \). This condition equals to \( 2\tau\sqrt{N(-3(\tau - \tau)(c_b - g) + \tau^2 N)} < 3(\tau - \tau)(c_b - g) - 2\tau^2 N \). Hence, if \( 3(\tau - \tau)(c_b - g) - 2\tau^2 N > 0 \), the condition can be rewritten as \( 4\tau^2 N(-3(\tau - \tau)(c_b - g) + \tau^2 N) < (3(\tau - \tau)(c_b - g) - 2\tau^2 N)^2 \) that further reduces to \( (\tau - \tau)^2 c_b(-3(c_b - g) + 4\tau N) < 0 \). In order for this to hold, we need to have \(-3(c_b - g) + 4\tau N < 0 \) and \(3(\tau - \tau)(c_b - g) - 2\tau^2 N > 0 \). In addition, we need to have \( c_b < g + \frac{\pi^2 N}{4(r - \tau)} \) as discussed in Lemma 1. In summary, we need to have the following satisfied in order for the derivative to be positive.
\[ -3c_b - 3g + 4rN < 0, \quad (16) \]
\[ 3(c_b - g)(\tau - r) - 2\tau r N > 0, \quad (17) \]
\[ c_b < g + \frac{\tau^2 N}{4(\tau - r)}. \quad (18) \]

Conditions 16 and 17 are satisfied at the same time if \((\tau - 4r)(3\tau - 4r) > 0\). On the other hand, conditions 17 and 18 are both satisfied when \(3\tau > 8r\). However given \(3\tau > 8r\), the condition 
\[(\tau - 4r)(3\tau - 4r) > 0\] reduces to \(\tau > 4r\). In addition, condition 17 becomes redundant since we have \(\tau > 4r\) and \(c_b < g + \frac{\tau^2 N}{4(\tau - r)}\) at the same time. In summary, in order for \(d(\tau)\) to be positive, we need to have \(\tau > 4r\) and \(c_b < g + \frac{\tau^2 N}{4(\tau - r)}\).

Proof of Proposition 4

Derivative of the equilibrium value of HP \(i\) with respect to the upper bound parameter is always negative given that we have \(c_b < g + \frac{\tau^2 N}{4(\tau - r)}\) from Lemma 1. Hence, part (a) of the proposition.

On the other hand, the derivative of the equilibrium value of HP \(i\) with respect to the lower bound parameter equals to
\[
\frac{N_0}{18(\tau - r)^2} \left( \frac{\tau^2 N_0}{3(\tau - r)(c_b - g)} + 2\tau^2 N_0 - 3(\tau - r)(c_b - g) \right).
\]
Because we have \(c_b < g + \frac{\tau^2 N}{4(\tau - r)}\) from Lemma 1, the sign of the derivative is positive if and only if \(r_i > \frac{\tau}{3}\). Hence, part (b) of the proposition.

Proof of Lemmas 4, and 5

The objective functions of the HIE provider and HPs are given as:
\[
\max_{s, p,b,v} \; \; p_b \mathbb{E}[N_v] + p_b \mathbb{E}[N_b] + g \mathbb{E}[N_b] - c_b \mathbb{E}[N_b] - c_v \mathbb{E}[N_v]s^2
\]
\[
\max_{q,b,i} \; \; q_b \left[ r_i \mathbb{E}[N_b] - p_b \right] + q_v \left[ r_i s \mathbb{E}[N_v] d - p_v \right] \quad \forall i
\]

As discussed in the paper, \(p_b^* = r_i^* \mathbb{E}[N_b | p_b^*]\), and \(p_v^* = r_v^* s^* \mathbb{E}[N_v | p_v^*] d\). If the net benefit from base HIE services is positive, i.e., \(r_i \mathbb{E}[N_b | p_b^*] - p_b^* \geq 0\), HP \(i\) joins the HIE, i.e., \(q_{bi} = 1\). On the other
hand, if the net benefit from the value-added services is positive, i.e., \( r_i s^* E[N_v|p_v^*]d - p_v^* \geq 0 \), HP \( i \) may consider requesting value-added services. However, as discussed before, value-added services are offered only to the HPs that are in the HIE network. Hence, the overall value should be positive in such a case, i.e., \( r_i E[N_b|p_b^*] - p_b^* + r_i s^* E[N_v|p_v^*]d - p_v^* \geq 0 \).

On the other hand, note that, the HIE provider does not consider the decisions of each of the HPs separately. However, it considers the total number of HPs interested in the base HIE service, and the total number of HPs interested in value-added services. In addition, the order of HPs that are interested in the base HIE service, and value-added services are the same since the benefit parameters are proportional. Therefore, we can solve the game as if any HP can receive value-added services. If the number of HPs that request value-added services is less than the number of HPs in the HIE network, then the solution is valid. However, if this condition is not satisfied, the solution is different as we discuss in detail later.

After substituting the values of \( p_b, p_v, E[N_b], \) and \( E[N_v] \) discussed in the main paper into the objective functions of the parties, we can solve the game among the parties. Rather than the payment terms \( p_b \) and \( p_v \), we can utilize \( r_b \) and \( r_v \) as decision variables because of the one-to-one correspondences between these variables and the payment terms as discussed in the main paper. In addition, the HIE provider needs to also decide the level of value-added service level \( s \). Hence, first order conditions with respect to \( s, r_b, \) and \( r_v \) reveal the equilibrium values of the variables below. Hence, Lemma 4.

\[
\begin{align*}
s &= \frac{N r_v^* (\tau - r_v^*) d}{2 c_v (\overline{r} - \tau)}, \\
 r_b^* &= \frac{2 \tau N - \sqrt{N \left( \overline{r}^2 N + 3 c_b (\overline{r} - \tau) \right)}}{3 N}, \\
 r_v^* &= \frac{2 \tau d - \sqrt{N d \left( \overline{r}^2 N d + 3 c_v s (\overline{r} - \tau) \right)}}{3 N d}.
\end{align*}
\]
After solving the above equations simultaneously, second order conditions imply that the following service level is optimal and the equilibrium number of HPs in the HIE, and those who request value-added services are:

\[ s^* = \frac{3\tau^2Nd}{25c_v(r-\bar{r})}, \]

\[ N_b = \min \left\{ \frac{N\bar{r} + \sqrt{N\left(N\bar{r}^2 - 3(r-\bar{r})(c_b-g)\right)} - 3(r-\bar{r})(c_b-g)}{3(r-\bar{r})}, N \right\}, \]

\[ N_v = \min \left\{ \frac{3\tau N}{5(r-\bar{r})}, N \right\}. \]

As discussed earlier, this solution is valid only if the number of HPs in the network is more than the number of HPs that request value-added services, i.e., if \( \frac{N\tau + \sqrt{N\left(N\tau^2 - 3(r-\bar{r})(c_b-g)\right)} - 3(r-\bar{r})(c_b-g)}{3(r-\bar{r})} \geq \frac{3\tau N}{5(r-\bar{r})} \). It is easy to show that this condition is true if and only if \( c_b < g + \frac{3\tau^2 N}{25(r-\bar{r})} \). Hence, if \( c_b > g + \frac{3\tau^2 N}{25(r-\bar{r})} \), there are HPs interested in value-added services but do not find base HIE services beneficial. In such a case, some HPs (or all of them) decide to join the HIE. Unfortunately, getting an analytical expression for the resultant network size is not possible due to the complexity of the equations in the solution of the equations. We do not provide these equations for brevity and clarity. However, we provide the conditions when value-added services increase the network size in parts (a) and (b) of Lemma 5, together with the bounds on the network size and the number of HPs that request value-added services.

**Proof of Lemma 6**

Optimal fees are derived by substituting the solution discussed in the proofs of Lemmas 4, and 5 into equations \( p_b^* = r_b^*\mathbb{E}[N_b|p_b^*] \), and \( p_v^* = r_v^*s^*\mathbb{E}[N_v|p_v^*]d \). Optimal level of value-added services is derived in the proofs of Lemmas 4, and 5.
Proof of Proposition 5

The results follow by taking the derivative of $N_b$ and $N_v$ (presented in part (d) of Lemma 5) with respect to $\tau$ and $\nu$.

Proof of Lemma 7

Equilibrium values of the parties are derived by substituting the solution discussed in the proofs of Lemmas 4, and 5 into the objective functions of the parties.

Proof of Proposition 6

We provide the proof only for part (a) of the proposition. The proof for part (b) is trivial, hence omitted. At the equilibrium, derivative of the value of HP $i$ with respect to $\tau$ equals to:

$$-rac{16\tau^2 N d^2}{11250(\tau - \bar{\tau})^3 N (\tau^2 N - 3(\tau - \bar{\tau})(c_b - g)) (4\tau - 2\tau_i - 5(\tau - 3\tau_i))}
- \frac{625c_v(\tau - \bar{\tau})}{11250(\tau - \bar{\tau})^3 N (\tau^2 N - 3(\tau - \bar{\tau})(c_b - g)) c_v}
\left(2 \left(\sqrt{N (\tau^2 N - 3(\tau - \bar{\tau})(c_b - g))} + \tau N\right) - 3(c_b - g)(\tau - \bar{\tau}) (\tau - 2\tau_i + 3\tau_i)\right).$$

This value is positive if and only if $\gamma$ that is defined below is positive:

$$\gamma = -16\tau^2 N d^2 \left(\sqrt{N (\tau^2 N - 3(\tau - \bar{\tau})(c_b - g))} (4\tau - 2\tau_i - 5(\tau - 3\tau_i))
- \frac{625c_v(\tau - \bar{\tau})}{11250(\tau - \bar{\tau})^3 N (\tau^2 N - 3(\tau - \bar{\tau})(c_b - g)) c_v}
\left(2 \left(\sqrt{N (\tau^2 N - 3(\tau - \bar{\tau})(c_b - g))} + \tau N\right) - 3(c_b - g)(\tau - \bar{\tau}) (\tau - 2\tau_i + 3\tau_i)\right).$$

If $r_i \leq \frac{g}{3}\tau$, $\gamma$ is always negative. This is because in order to have a partial network where some but not all HPs request value-added services, we need to also have $c_b < g + \frac{3\tau^2 N}{2(\tau - \bar{\tau})}$. Hence, part (a1) of the proposition.

In order to prove part (a2) of the proposition, let us give an example when the value of HP $i$ increases with $\tau$. First set $\bar{\tau} = 0$ in order to have high heterogeneity, and rewrite $\gamma$ as:

$$\tau^2 \left(-16\tau N d^2 \sqrt{\tau N (\tau N - 3(c_b - g))} (4\tau - 5r_i)
- \frac{625c_v(\tau - \bar{\tau})}{2\tau \left(\sqrt{\tau N (\tau N - 3(c_b - g))} + \tau N\right) - 3(c_b - g)(\tau + 3r_i)\right).$$

Observe that, if $r_i > \frac{g}{6}\tau$ and if $d > \frac{25\sqrt{c_v(3(c_b - g)(\tau + 3r_i) - 2\tau \left(\sqrt{\tau N (\tau N - 3(c_b - g))} + \tau N\right))}}{9\tau \sqrt{\tau N (\tau N - 3(c_b - g)) (5r_i - 4\tau)}}$, value of HP $i$ increases with $\tau$. Hence, part (a2) of the proposition.