REQUEST: A Query Language for Customizing Recommendations

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Abstract

Initially popularized by Amazon.com, recommendation technologies have become widespread over the past several years. However, the types of recommendations available to the users in these recommender systems are typically determined by the vendor and therefore are not flexible. In this paper we address this problem by presenting the recommendation query language REQUEST that allows users to customize recommendations by formulating them in the ways satisfying personalized needs of the users. REQUEST is based on the multidimensional model of recommender systems that supports additional contextual dimensions besides classical User and Item dimensions and also OLAP-type aggregation and filtering capabilities. The paper also presents a recommendation algebra, shows how REQUEST recommendations can be mapped into this algebra, and analyzes the expressive power of the query language and the algebra. Finally, the paper shows how users can customize their recommendations using REQUEST queries through a series of examples.

Keywords: personalization, recommender systems, recommendation query language, recommendation algebra.
1. Introduction

Recommender systems represent an important class of personalization technologies that help users to deal with information overload in e-commerce and numerous other applications. There has been much work done in the area of recommender systems over the past decade since the introduction of the first papers on the subject [12, 19, 20], especially after these technologies were popularized by Amazon and Netflix, as well as after the establishment of the $1,000,000 Netflix Prize Competition that attracted over 24,000 contestants from 154 countries [6]. A recent survey of the rapidly growing field of recommender systems can be found in [3].

Most of the work in recommender systems focuses on a two-dimensional paradigm of recommending items to users or users to items (e.g., books to customers or buyers for books). Although there are different types of approaches to deriving recommendations, including the ranking- [10] and market-basket-analysis-based [16], the majority of the academic work in recommender systems and implementations of commercial systems, including Amazon and Netflix, focuses on the rating-based approach [3], where recommendations use explicit or implicit ratings provided by the end-users.

Rating-based approaches are usually classified into content-based, collaborative, and hybrid [7]. In content-based recommendation methods, rating $R(u,i)$ of item $i$ for user $u$ is typically estimated based on the ratings $R(u,i')$ assigned by the same user $u$ to other items $i'$ that are “similar” to item $i$ in terms of their content. For example, in order to recommend movies to user $u$, the content-based approach tries to understand user preferences by analyzing commonalities among the content of the movies user $u$ has rated highly before. Then, only the movies that have a high degree of similarity with customer’s past preferences are recommended.

Collaborative recommender systems try to predict rating $R(u,i)$ of item $i$ for user $u$ based on how other “similar” users $u'$ previously rated item $i$. Here “user similarity” is defined in terms of
the distance between the ratings users \( u \) and \( u' \) assigned to the items that both of them rated, the most popular types of distance metrics being correlation- and cosine-based measures between two rating vectors [3]. Then collaborative filtering methods recommend those items to the user that she has not rated yet and that were highly rated by the similar users.

Content and collaborative methods can be combined into a hybrid approach in several different ways [7]. One popular way to combine them is by learning and maintaining user profiles based on the content analysis of the items preferred by the users, and then directly comparing the resulting profiles to determine similar users in order to make collaborative recommendations. Other types of hybrid methods are also possible and are described in [3].

Although the traditional two-dimensional user/item paradigm described above is suitable for some applications, such as recommending books and music CDs, it is significantly less suitable for the “context-rich” applications, such as traveling or shopping applications. For example, when recommending vacations to travelers, one would likely recommend a different vacation to a customer in the winter than in the summer, i.e., the time-of-travel context is clearly important when making recommendations. Similarly, when recommending groceries, a “smart” shopping cart [23] needs to take into account not only information about products and customers, but also such information as shopping date/time, store, who accompanies the primary shopper, products already placed into the shopping cart, and its location in the store. Clearly, the two-dimensional paradigm of classical recommender systems is less suitable for these applications.

To provide better recommendations in such “contextually rich” applications, one may need to consider other dimensions besides Item and User. For example, when Netflix or any on-demand movie provider recommends movies, it may consider such additional dimensions as Time when the movie was seen, Company in which the movie was seen (e.g., alone, with friends, parents, etc.), and Place in which it was seen (e.g., in the theater or at home). A completely different
movie may be recommended by Netflix to a student when he wants to see it on a Saturday night with his girlfriend than when he wants to see it on Thursday evening with his parents at home. In [4, 5] we proposed a new multidimensional approach to recommender systems where we incorporated multiple dimensions and the OLAP-based cubes of ratings into the recommendation model. To estimate missing ratings in multidimensional cubes, we proposed the reduction-based method in [4] and the heuristic-based and model-based methods in [2].

However, the multidimensional approach described in [4] and the classical two-dimensional recommendation methods have one significant limitation in common. These methods are hard-wired by the developers into the recommender systems, are inflexible and limited in their expressiveness, and, therefore, neglect some possible needs of the users. For example, a typical recommender system would recommend top \( k \) items to a user, or the best \( k \) users for a product. This situation is quite limited, especially in the multidimensional settings, where the number of possible recommendations increases significantly with the number of dimensions [5]. Therefore, there is a need to empower end-users and other stakeholders by providing them with the tools for expressing recommendations that are of interest to them. For example, Jane Doe may need a recommendation for the best two dates to go on vacation to Jamaica with her boyfriend. Also, Netflix or an on-demand movie service, such as provided by the Time Warner Cable, can envision a web-based interface to a multidimensional cube of ratings that lets the users express the recommendations that are of interest to them or automatically tailors recommendations based on a given context, such as time of day or day of week. For example, a certain user may seek recommendations for him and his girlfriend of top 3 movies and the best times to see them over the weekend, and he enters this request into the recommender system via the web-based interface. Such query-based recommendation applications are not limited to on-demand movies but are relevant to a broad range of recommendation applications, including retailing, financial, travel and
other applications. Furthermore, we believe that flexible recommendation capabilities would be appealing to a variety of different users, and not just to the end-users who are direct recipients of recommendations. For example, such functionality would be useful to analysts of the company that is providing recommendation services, who may want to take advantage of all the knowledge that their recommender system holds and analyze it from a variety of different perspectives (“show me the top 2 movie genres for each user age bracket”, etc.).

One tool for expressing such requests is a recommendation language that is similar to how database users use query languages to retrieve information from databases. In fact, one may try to use SQL for this purpose, and the above recommendation for Tom can be expressed in SQL as

```
SELECT R.MovieId, R.TimeId, R.UserId, R.CompanionId, AVG(R.PersonalRating)
FROM MovieRecommender R, User U, Time T, Companion C
WHERE R.UserId = U.UserId AND R.TimeId = T.TimeId AND R.CompanionId = C.CompanionId
AND U.Name = “Tom” AND T.WeekTime = “Weekend” AND C.Type = “Girlfriend”
GROUP BY R.MovieId, R.TimeId, R.UserId, R.CompanionId
```

where User and Companion are the relations storing information about customers and different types of companions, MovieRecommender is the ratings table and Time is the temporal dimension table. Although “doable,” this SQL query and, more generally, SQL at large would have the following problems when used for recommendation purposes. First, notice that SQL does not exactly provide the requested recommendation: it returns the list of tuples (movies, times to see them, users, etc.) and does not specify what is recommended to whom and does not provide the top 3 recommended movie/time pairs. More generally, as it will be shown in the paper, there are certain recommendations that cannot be expressed in SQL (but can in the language proposed in this paper). Second, SQL is a comprehensive, general-purpose query language and, therefore, many of the possible SQL queries do not represent recommendations. Therefore, in order to help the end-user formulate recommendations correctly and meaningfully, one may want to impose elaborate constraints on SQL to be able to restrict the language for the recommendation task. We
have studied this issue and realized that it is very hard to develop a simple, elegant and intuitive system of such constraints. A better alternative would be to introduce a language that is directly defined on top of the “native” multidimensional recommendation model. Third, the above SQL query is fairly cumbersome: it constitutes a join of four relational tables, has 6 conditions in the WHERE clause, has the GROUP BY statement and the aggregation function AVG. Clearly, there should be better, more intuitive ways to express this simple type of recommendation, and this served as a direct motivation for this research. Fourth, this cumbersomeness may have not only a cognitive effect on the users writing queries, but could possibly also affect performance in some cases, since processing multiple join queries can be a very time-consuming operation. In summary, the above issues can be attributed to the task and model mismatch. SQL is a comprehensive query language, which makes it a less intuitive tool for users in a highly specialized application domain, and being a general-purpose language SQL may not have some specialized capabilities important for recommender systems. Also, SQL is based on the relational data model and multidimensional recommendations on the multidimensional model [4], which needs to be mapped into the relational model to support SQL queries. To avoid these issues, it is advantageous to develop a recommendation language based on the characteristics of the application domain and the task at hand (using the multidimensional model, etc.), as we do in the paper.

To provide flexible and user-driven recommendations and to address the previously specified issues with using SQL as a recommendation language, we designed a new recommendation query language REQUEST\(^1\), which allows its users to express in a flexible manner a broad range of recommendations that are tailored to their own individual needs and, therefore, more accurately reflect their interests. For example, the earlier recommendation for Tom can be expressed in

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\(^1\) REQUEST is an acronym for REcommendation QUEry STatements.
REQUEST as

\[
\text{RECOMMEND Movie, Time TO User, Companion}
\]
\[
\text{USING MovieRecommender}
\]
\[
\text{BASED ON PersonalRating}
\]
\[
\text{RESTRICIT User.Name = “Tom” AND Time.TimeOfWeek=“Weekend” AND Companion.Type = “Girlfriend”}
\]
\[
\text{SHOW TOP 3}
\]

where MovieRecommender is a 5-dimensional cube of ratings having dimensions User, Movie, Time, Companion, and Theater; also, PersonalRating represents the ratings measure for the cube.

The above REQUEST query is based on the OLAP paradigm [9], which is a natural choice for querying multidimensional recommender systems since the data model of REQUEST matches the multidimensional data model of the ratings cube. Besides REQUEST, we also present a multidimensional recommendation algebra that is used for defining certain “core” parts of REQUEST queries. We also describe how these core REQUEST queries can be processed by mapping them into this algebra.

The initial version of our recommendation query language, called RQL, was introduced in an earlier workshop paper [5], where only the preliminary ideas of how to define the query language were presented using a few examples. In this paper, we systematically redesigned the language by formally introducing its syntax, semantics, and the corresponding recommendation algebra. This allowed us to significantly extend capabilities of the language over its preliminary version [5]. To reflect these major changes, we renamed the language from RQL to REQUEST.

This paper makes the following contributions. It proposes language REQUEST for expressing flexible user-driven recommendations and presents its syntax and semantics. It also presents recommendation algebra RA, which enhances the systematic definition of REQUEST. We also show how the core REQUEST queries can be mapped into RA, thus providing a way to process these queries, and provide the analysis of the expressive power of REQUEST and RA.
2. Background: Multidimensional Recommender Systems

A multidimensional ratings cube is defined as a tuple \((D, M, H, E, L)\) as follows.

**Dimensions (D).** \(D = \{d_1, d_2, \ldots, d_n\}\) is a set of \(n\) dimensions, where each \(d_i\) is a dimension name.

For example, in addition to the standard User and Movie dimensions of the traditional movie recommender systems, such as MovieLens [17], we consider other contextual dimensions [4, 5], such as Time, Theater and Companion., i.e., \(D = \{\text{User, Movie, Time, Theater, Companion}\}\).

**Attribute Hierarchies (H).** Each dimension \(d_i\) is represented by a set of attributes \(A_i=\{a_{i1}, \ldots, a_{it}\}\) where each \(a_{ij}\) is an attribute name; e.g. \(A_{\text{time}}=\{\text{Date, DayOfWeek, TimeOfWeek, Month, Quarter, Year}\}\). The domain of attribute \(x\) of dimension \(d\) is denoted as \(\text{dom}(d,x)\), e.g., \(\text{dom}(\text{Time.DayOfWeek}) = \{\text{Mon, Tue, Wed, Thu, Fri, Sat, Sun}\}\).

The multidimensional recommendation model allows for OLAP-based aggregation hierarchies [4, 5] that help aggregate ratings according to the methods described in [4]. In particular, attributes \(A_i\) of dimension \(d_i\) form a directed acyclic graph (i.e., a hierarchy) \(H_i = (V_i, E_i)\) with set of nodes \(V_i\) (where \(V_i = A_i\)) and set of edges \(E_i\). There exists a directed edge in \(H_i\) from attribute \(x \in A_i\) to attribute \(y \in A_i\), iff every value of \(x\) uniquely determines the value of \(y\), i.e., if attribute \(y\) is functionally dependent on attribute \(x\). Such an edge will be denoted \((x, y)\) or \(x \rightarrow y\). We will assume that \(H_i\) has a single root node, \(\text{Root}(H_i)\), which we will call the key dimension attribute, consistent with the standard database terminology. Let \(H = \{H_1, \ldots, H_n\}\).

Given hierarchy \(H_i\) and attribute \(d_{i,x} \in A_i\), we define \(\text{SubGraph}(H_i, d_{i,x})\) to be a subgraph of \(H_i\) rooted at \(d_{i,x}\), i.e., it defines the graph containing all the nodes and edges reachable from \(d_{i,x}\).

**Elements (E).** Each dimension \(d_i\) in a cube is represented by a set of elements \(E_i\). For instance, dimension Movie in our example is represented by all the movies available for the users to rate. For simplicity, and without loss of generality, we use the domain of the key dimension attribute to represent the set of elements of \(d_i\), i.e., \(E_i := \text{dom}(\text{Root}(H_i))\). An example of the elements’ set for
the User dimension would be a set of all user IDs available in the data. Let \( E = \{ E_1, \ldots, E_n \} \).

**Measures (M).** \( M = \{ m_1, m_2, \ldots, m_k \} \) represents a set of measures, where each \( m_i \) is a different type of a rating from domain \( \text{dom}(m_i) \). The measures can either be numeric or Boolean. A numeric measure usually represents a discrete finite ordered value, e.g., a movie rating on the scale of \( \{1, \ldots, N\} \). A Boolean measure can be used to represent a “status flag” denoting a certain state of a rating or a specific context, e.g., indicating whether a given movie has been seen by a given user.

**Example 1.** Consider the application for recommending movies to users that has the following dimensions, each dimension defined by the attributes specified in parentheses:

- **Movie:** the set of all the movies that can be recommended; it is defined as Movie(MovieID, Title, Length, ReleaseYear, Director, Genre).
- **User:** the people to whom movies are recommended; it is defined as User(UserID, Name, Address, Age, Gender, Profession).
- **Theater:** the movie theaters showing the movies; it is defined as Theater(TheaterID, Name, Address, Capacity, City, State, Country).
- **Time:** the time when the movie can be or has been seen; it is defined as Time(Date, DayOfWeek, TimeOfWeek, Month, Quarter, Year).
- **Companion:** represents a person or a group of persons with whom one can see the movie. It is defined as Companion(companionType), where attribute companionType has values “alone”, “friends”, “girlfriend/boyfriend”, “family”, “co-workers”, and “others”.

We also use three rating measures in this example: PublicRating, a numeric measure specifying how much the general public liked the movie; PersonalRating, a numeric measure specifying how much a particular person liked or is predicted to like the movie in the settings specified by the Time, Theater, and Companion dimensions; and Consumed, a Boolean measure specifying whether or not a given user has actually seen a given movie in a given context. The PersonalRating assigned to a movie by a person depends on where and how the movie has been seen, with whom and at what time. For example, the type of movie appropriate for a specific college student may differ depending upon whether the movie is part of a romantic weekend date with her boyfriend or a weekday study-break with visiting friends and parents. Finally, we consider the following aggregation hierarchies: **Movie:** MovieID \( \rightarrow \) Genre; **User:** UserID \( \rightarrow \) Age, UserID \( \rightarrow \) Gender, UserID \( \rightarrow \) Profession; **Theater:** TheaterID \( \rightarrow \) City \( \rightarrow \) State \( \rightarrow \) Country; **Time:** Date \( \rightarrow \)
DayOfWeek → TimeOfWeek, Date → Day → Month → Quarter → Year.

**Cube Cells (L).** Each cube essentially is a *partially defined* rating function $R$ from an $n$-dimensional space of $E_1 \times \ldots \times E_n$ to a $k$-dimensional space of measures, i.e., $R: E_1 \times \ldots \times E_n \rightarrow \text{dom}(m_1) \times \ldots \times \text{dom}(m_k)$. Alternatively, a cube can be perceived as a set of cells $L$, each cell $l \in L$ consisting of the tuple $(\text{address}, \text{content})$, i.e., $l = (\text{address}, \text{content})$, where $\text{address} = (\alpha_1, \ldots, \alpha_n)$, $\alpha_i \in E_i$, and $\text{content} = (\beta_1, \ldots, \beta_k)$, $\beta_i \in \text{dom}(m_i)$. Since the mapping $R$ is partial, $\text{content}$ can also have value $\text{NULL}$ for some cells. We also use the notation $L[\text{address}] = \text{content}$ to refer to a specific cell, and $L[\text{address}],m_j$ to refer to a specific measure within a cell. Furthermore, the ratings $R(\alpha_1, \ldots, \alpha_n)$ of the recommendation space $S = E_1 \times E_2 \times \ldots \times E_n$ are either explicitly provided by the users or are implicitly inferred by the system as described below. For example, $R(\text{Aviator}, \text{Jane}, \text{theater5}, 2/19/2005, \text{boyfriend}) = (6, 8, \text{True})$ means that Jane gave rating 6 (i.e., PersonalRating = 6) to “Aviator” that she actually saw (i.e., Consumed = True) with her boyfriend on February 19, 2005 in movie theater 5, but the general public gave the movie the rating of 8 (i.e., PublicRating = 8).

Given these preliminaries, the recommendation problem is defined as follows. First, the system needs to estimate the unknown ratings and make the rating function $R$ total [4]. Second, to make a recommendation, one needs to select certain *non-overlapping* “what” dimensions $d_{i_1}, \ldots, d_{i_k}$ ($k < n$) and certain “for whom” dimensions $d_{j_1}, \ldots, d_{j_l}$ ($l < n$) and, accordingly, recommend for each tuple $(\alpha_{j_1}, \ldots, \alpha_{j_l}) \in E_{j_1} \times \ldots \times E_{j_l}$ tuple $(\alpha_{i_1}, \ldots, \alpha_{i_k}) \in E_{i_1} \times \ldots \times E_{i_k}$ maximizing the rating $R(\alpha_1, \ldots, \alpha_n)$ across all the tuples $(\alpha_1, \ldots, \alpha_n)$ coinciding with $(\alpha_{j_1}, \ldots, \alpha_{j_l}) \in E_{j_1} \times \ldots \times E_{j_l}$ on corresponding dimensions $d_{j_1}, \ldots, d_{j_l}$.

Since the rating cube is only partially filled, it is important to estimate the unspecified ratings for recommendation purposes. This multidimensional rating estimation problem is addressed in
[4], where the reduction-based method of estimating unknown ratings in terms of the known ratings is presented. To understand how it works, assume that we want to recommend a movie to Jane Doe who wants to see it with her boyfriend on a Saturday night in a movie theater. If the Time dimension is partitioned into weekend and weekday components and since Saturday night falls on a weekend, the reduction-based approach uses only the ratings for the movies seen on weekends by customers with their boyfriends/girlfriends in the movie theaters in order to provide recommendations for Jane Doe. It was shown that this approach outperforms the standard collaborative filtering in multidimensional settings under certain conditions [4]. Alternative multidimensional rating estimation methods include heuristic- and model-based approaches [2].

In this paper we focus on the querying capabilities of the REQUEST language and, therefore, we assume that the multidimensional ratings cube is fully pre-computed before users start issuing recommendation queries. In other words, we assume that all the unknown ratings have been estimated using any of the aforementioned rating estimation techniques. How to perform rating estimation “on demand” based on the query that was issued on a partially filled ratings cube constitutes an interesting future research problem, as we mention in Section 6.

The work described in [4] focuses on presenting the multidimensional recommendation model and does not specify how to express a wide variety of recommendations that are possible in multidimensional settings. In the next section we address this limitation by presenting the query language REQUEST for expressing such recommendations.

3. Recommendation Query Language REQUEST
In this section, we describe the language by providing various examples of REQUEST queries in Section 3.1, then present its syntax in Section 3.2 and semantics in Section 3.3.

3.1. Introducing REQUEST via Examples
All examples presented in this section are based on the 5-dimensional MovieRecommender cube of ratings having the schema from Example 1. The first example presents the most basic and
traditional recommendation request.

**Query 1:** Recommend the best movies to users:

RECOMMEND Movie TO User
USING MovieRecommender
BASED ON PersonalRating

In this query, the RECOMMEND and TO clauses specify that movies will be recommended to users. The USING clause specifies the name of the multidimensional rating cube. The BASED ON clause specifies that personal ratings are used for the recommendation purposes. The movies in this query are ordered separately for each user based on the PersonalRating measure that is either provided by the user or estimated from the set of known ratings as mentioned in Section 2. The query returns the highest-rated movie for each user. Query 1 actually uses some defaults, and the equivalent query with the *explicitly* specified parameters is:

RECOMMEND Movie (MovieID) TO User (UserID)
USING MovieRecommender
BASED ON PersonalRating(AVG)
SHOW TOP 1 BY PersonalRating

Qualifier AVG specifies that, when the MovieRecommender cube is reduced to two dimensions Movie and User, all the ratings of a movie seen by a user on different occasions (note that the user could see or rate the same movie more than once, i.e., in different contexts), are aggregated by averaging their values. Each measure can have its own default aggregation function (e.g., AVG in this case). The SHOW TOP \( k \) clause returns \( k \) best movies for the user ordered by aggregated PersonalRating measure. By default, \( k = 1 \), i.e., we did not want recommendation queries returning a huge number of movies for each user by default (even if the movies are sorted by their relevance to each user) without end-user explicitly asking for it. MovieID and UserID represent the dimensional attributes that will be used when displaying the results.

Next we introduce the restrictions on the recommendation criteria.
Query 2: Recommend, using personal ratings, top 5 action movies to users older than 18.

RECOMMEND Movie TO User
USING MovieRecommender
RESTRICT Movie.Genre = "Action" AND User.Age >= 18
BASED ON PersonalRating(AVG)
SHOW TOP 5 BY PersonalRating

The RESTRICT clause is used to select the movies and the users satisfying the selection criteria. Then only the selected movies are ordered for each selected user based on the instructions specified in the BASED ON and the SHOW clauses, as discussed above.

Note that the syntax of REQUEST differs from that of SQL. This is done on purpose to reflect significant differences between these languages. This is discussed further in Section 3.2.

We next show how ratings are filtered using the POSTFILTER clause.

Query 3: Recommend top 5 movies to the user for the weekend but only if personal ratings of the movies are higher than 7 (if fewer than 5 movies satisfy these criteria, then show only those satisfying them).

RECOMMEND Movie TO User
USING MovieRecommender
RESTRICT Time. TimeOfWeek = "Weekend"
BASED ON PersonalRating(AVG)
POSTFILTER PersonalRating > 7
SHOW TOP 5

Query 3 demonstrates that different clauses (RESTRICT and POSTFILTER) are used for the selections of attributes and ratings. First, only the weekend ratings are selected with the RESTRICT clause. Then they are aggregated using the “BASED ON PersonalRating (AVG)” clause. Only then the POSTFILTER clause is applied to these aggregated PersonalRatings and only those greater than 7 are selected. If we want to restrict non-aggregated ratings, we should use the PREFILTER clause, as is done in Query 5. The reasons for using separate RESTRICT and POSTFILTER clauses when restricting attributes and ratings are discussed in Section 3.2.

The next example shows that more than one dimension can be used in recommendations, i.e., Movie and Time are recommended to User and Companion.
**Query 4:** Recommend to Tom and his girlfriend top 3 movies and the best times to see them over the weekend.

RECOMMEND Movie, Time TO User, Companion
USING MovieRecommender
RESTRICT User.Name = “Tom” AND Time.TimeOfWeek =”Weekend” AND Companion.Type = “Girlfriend”
BASED ON PersonalRating
SHOW TOP 3

Sometimes, a certain group of people may be interested in a certain type of movies. The next example shows how this type of aggregation can be done in REQUEST.

**Query 5:** Recommend movie genre to various professions using only the movies with personal ratings bigger than 6:

RECOMMEND Movie.Genre TO User.Profession
USING MovieRecommender
PREFILTER PersonalRating > 6
BASED ON PersonalRating(AVG)

This query aggregates rating scores for individual movies into averaged rating scores for different genres of movies. Also, individual users are aggregated by profession, and each profession becomes a new target for a recommendation. Before the ratings are aggregated, the PREFILTER operator selects the ratings bigger than 6, and only these ratings are aggregated. It differs from the POSTFILTER operator in Query 3 since it deals with non-aggregated ratings. This distinction is crucial in some recommendation settings. For example, in some cases the end-user may want to generate aggregate recommendations (e.g., which individual movies are most liked by an average user?) based on the entire set of ratings, and only then filter out non-highly recommended items – the user should use POSTFILTER in such cases. In other cases, however, the user may want to remove the low ratings first (i.e., remove the obvious “haters” of a particular item) and only then aggregate the ratings to obtain the recommendations – the user should use PREFILTER for this.

The next example demonstrates that recommendations are not restricted to the User dimension; in general, different things can be recommended to various objects.
Query 6: Identify the top two professions that appreciate the movie “Beautiful Mind” the most.

RECOMMEND User.Profession TO Movie
USING MovieRecommender
RESTRICT Movie.Title = “Beautiful Mind”
BASED ON PersonalRating(AVG)
SHOW TOP 2

Remember that a rating score for a movie typically either is explicitly specified by the user or is estimated from the existing user-specified ratings using one of the rating estimation methods described in [2, 4]. The next query is a modification of Query 1 that makes use of this fact.

Query 7: Recommend best movies to users that they have not seen yet.

RECOMMEND Movie TO User
USING MovieRecommender
BASED ON PersonalRating(AVG), Consumed(DISJ)
POSTFILTER NOT(Consumed)
SHOW TOP 1 BY PersonalRating

This query collects all the ratings given to a movie by a user (note that the user can provide multiple ratings to a movie, seen in different contexts). Consumed is a Boolean flag related to PersonalRating measure specifying whether a movie was seen (“consumed”) by a user on certain occasion. Consumed(DISJ) is the disjunction of the values of all of these flags for a movie/user pair. If this disjunction is true (the cumulative Consumed flag is True), this means that on at least one occasion, the user has seen the movie. The POSTFILTER NOT(Consumed) statement removes these cases. Thus, only the movies that the user has not seen before are recommended.

The next query shows how recommendations based on multiple ratings are used.

Query 8: Show top 5 movies with both public ratings and personal ratings bigger than 8 to students based only on the movies they have seen.

RECOMMEND Movie TO User
USING MovieRecommender
RESTRICT User.Profession = “Student”
PREFILTER Consumed
BASED ON PersonalRating(AVG), PublicRating(AVG)
POSTFILTER PublicRating > 8 AND PersonalRating > 8
SHOW TOP 5 BY PersonalRating, PublicRating
This query first selects the ratings of movies provided by students that they have previously seen (i.e., prefilters them based on the Consumed flag). Then it aggregates them based on personal and public ratings and selects only PublicRating and PersonalRating that on average are greater than 8. Finally, it sorts the movies for each user based on these two ratings and selects the top five of them for each user.

Note that, since in a recommender system ratings can be provided by users or estimated by software, for any rating measure we have an option of differentiating between actual, estimated, and other types of ratings. The REQUEST language supports this functionality via binary flags (implemented as separate Boolean measures) that can be used in PRE- and POSTFILTER clauses, as well as be aggregated using specific Boolean aggregation functions. For example, Queries 7 and 8 use the Consumed flag specifying if the rating is based on the movie that the user has actually seen. This is possible because, as explained earlier, all the estimated ratings and the related flags are precomputed, and thus can be used conceptually as additional measures.

After introducing REQUEST via examples, we next define the syntax of the language.

3.2. Syntactic Definition of REQUEST
The BNF specification of REQUEST syntax is presented in Figure 1. First, note that we do not mimic the syntax of SQL for REQUEST because, if we tried to do so, this would likely cause many false assumptions on behalf of the users who may assume that properties of SQL operators automatically extend to REQUEST simply because the names of the operators are the same. For example, the RESTRICT clause of REQUEST is significantly more restrictive than the WHERE clause of SQL, as will be explained below. This observation is also applicable to various other REQUEST clauses that will be discussed later in this section.

We have been developing REQUEST as a domain-specific query language (i.e., a language for a specialized application domain – recommender systems) as opposed to a general language for
querying data. Following this approach, we tried to make sure that every construct of the language has a well-defined/intuitive meaning from recommendation point of view, while at the same time trying to maintain expressiveness and rigor of the language.

In particular, as Figure 1 shows, recommendations are restricted to a single cube of ratings (i.e., only one cube is allowed in the USING clause), thus disallowing joins between cubes in REQUEST. This is the case, because multi-cube recommendations seldom have meaningful and practically important applications and can lead to various complications and side-effects. For example, in order to join two cubes on a certain dimension, the two dimensions should be identical for all the levels of the aggregation hierarchy. In particular, if two cubes are joined on Time, the Time scales should obviously be the same for both dimensions in order to facilitate the correct aggregation of the joined data. Also, if multiple cubes are used in queries, then, there is a dilemma of whether PUSH and PULL operators [1] that can “push” one of the dimensions to become a measure and also “pull” a measure as a new dimension should be supported at the algebraic level. Without such operators incorporated into REQUEST, some interesting queries either cannot be expressed or can be done only in a convoluted manner. However, incorporating the PUSH and PULL operators into the language creates numerous complications for REQUEST because dimensions and measures are very different semantically in the multidimensional recommendation model. For example, it is not clear what it means to push dimension User into the ratings in the context of recommender systems. Therefore, if we support multiple rating cubes within a single recommendation query, then we have problems both with and without the PUSH and PULL operators in REQUEST. Finally, when joining cubes, estimated ratings need to be re-evaluated for the joined cubes, often in significantly higher-dimensional spaces. This can lead to the well-known rating sparsity problem, i.e., when it becomes hard to estimate unknown ratings in high-dimensional spaces due to the lack of the known ratings.
// general syntax of a REQUEST query
REQUEST_query ::= RECOMMEND recommend_dim_list TO recipient_dim_list
  USING cube_name
  [ RESTRICT dimension_restrictions ]
  [ PREFILTER preaggregation_measure_restrictions ]
  BASED ON aggr_measure_list
  [ POSTFILTER postaggregation_measure_restrictions ]
  [ SHOW measure_rank_restriction ]

// RECOMMEND and TO clauses
recommend_dim_list ::= dimension_list
recipient_dim_list ::= dimension_list
dimension_list ::= single_dimension { , single_dimension }*
single_dimension ::= { dimension_name [ output_attribute_list ] | dimension_attribute }
output_attribute_list ::= ( attribute_name { , attribute_name } )*  

// USING clause
cube_name ::= variable

// RESTRICT clause
dimension_restrictions ::= single_dimension_restriction { AND single_dimension_restriction }*
single_dimension_restriction ::= dimension_attribute { numeric_comparison | textual_comparison | set_membership_test }

// BASED ON clause
aggr_measure_list ::= single_aggr_measure { , single_aggr_measure }*
single_aggr_measure ::= measure_name [ { rating_aggr_function } ]
rating_aggr_function ::= numeric_aggr_function | boolean_aggr_function
numeric_aggr_function ::= MIN | MAX | SUM | AVG
boolean_aggr_function ::= DISJ | CONJ | MAJORITY

// PREFILTER and POSTFILTER clauses
preaggregation_measure_restrictions ::= measure_restrictions
postaggregation_measure_restrictions ::= measure_restrictions
measure_restrictions ::= single_measure_restriction { logical_op single_measure_restriction }*
single_measure_restriction ::= numeric_measure_restriction | boolean_measure_restriction
logical_op ::= AND | OR
numeric_measure_restriction ::= measure_name numeric_comparison
boolean_measure_restriction ::= measure_name | NOT ( measure_name ) | measure_name = boolean_value

// SHOW clause
measure_rank_restriction ::= { TOP | BOTTOM } number [ BY measure_list ]
measure_list ::= measure_name { , measure_name }*

// common expressions
dimension_attribute ::= dimension_name . attribute_name
dimension_name ::= variable
attribute_name ::= variable
measure_name ::= variable
numeric_comparison ::= { = | <= | > | < | <= | >= } number
textual_comparison ::= { = | LIKE } 'string'
set_membership_test ::= { IN | NOT IN } { value_list }
value_list ::= numeric_value_list | textual_value_list
numeric_value_list ::= number { , number }*
textual_value_list ::= 'string' { , 'string' }*
boolean_value ::= true | false

Figure 1. BNF Specification of REQUEST Syntax.
In summary, we decided not to support joins in REQUEST queries for all the above reasons.

A better alternative to supporting joins in REQUEST is for the domain expert to manually build a single cube that is the join of two or more individual cubes. This would follow an approach that is similar to doing materialized joins in a relational database.

The RESTRICT clause contains dimension_restrictions that constitute the standard restrictions of the “slice-and-dice” operator in OLAP systems. Each individual restriction is limited to the numeric and textual comparison of a dimension attribute to a constant value (or a set of values), as specified in the BNF grammar above, and these dimensions and attributes have to be present in the schema of the cube_name cube. Moreover, multiple restrictions in a single RESTRICT clause are permitted, but only if combined by logical operator AND. Disjunctions (OR) are not allowed because the result of such restrictions would no longer be a multidimensional cube, as illustrated in Figure 2.

We also would like to note that, although the RESTRICT clause is somewhat similar to the WHERE clause of SQL, they also have the following key differences mostly stemming from our need to restrict REQUEST to make it more suitable for the recommendation applications. First, the WHERE clause of SQL is not limited to conjunctions, as RESTRICT is, but can also have disjunctions. Second, each individual restriction (conjaunct) in the RESTRICT clause can involve only one dimension (i.e., a comparison of some dimension attribute to a constant, as mentioned

![Figure 2](image-url)
earlier) in order to ensure that the result of the restriction is still a proper multidimensional cube. For this reason, for example, the restriction “RESTRICT User.Age > Movie.Length” is not allowed in REQUEST. In contrast, the WHERE clause of SQL allows having attributes from multiple relations in a single condition. Third, the WHERE clause of SQL supports nested queries, whereas RESTRICT does not. Besides these major differences between the two clauses, there are also minor differences apparent from the BNF grammars of the two languages.

The PREFILTER and POSTFILTER clauses contain measure_restrictions that constitute a set of restrictions on various types of measures used in cube_name. Note that, unlike dimension_restrictions, both AND and OR operators are allowed in measure_restrictions according to Figure 1. REQUEST uses separate RESTRICT and PRE-/POSTFILTER clauses when restricting attributes on dimensions and rating measures, as opposed to combining them into one clause. We do this for the following reasons. First, these two types of restrictions are semantically very different: the first one restricts the contextual information by imposing conditions on dimensional attributes, while the second does it on the measures. Second, the POSTFILTER clause must be kept separately because, unlike RESTRICT, it is applicable to the aggregate ratings, which are quite different from the un-aggregated ratings. Although this point is not applicable to the PREFILTER clause, it is better to keep both the PRE- and the POSTFILTER clauses (as they are symmetric), which makes it impossible to merge PREFILTER and RESTRICT clauses. Third, as pointed out before, only disjunctions are allowed in the RESTRICT, but both conjunctions and disjunctions are allowed in the PRE- and POSTFILTER clauses, making it important to treat them separately. Fourth, to keep the semantics of recommendations clear, it is important not to mix the rating measures and dimensional restrictions by prohibiting expressions of the form “PersonalRating > Time.DayOfWeek.” These were the reasons for keeping the two types of restrictions separately. Note that this situation is not unlike the case in temporal
databases, where separate WHERE and WHEN clauses are used for regular and temporal dimensions [21].

Formally, the output of a recommendation query is a set of tuples \( \{(t, L_t) \mid t \in T\} \), where \( t \) is a recommendation recipient and \( L_t \) is a list of recommendations for recipient \( t \). For example, in a movie recommender system, a simple example of a recommendation tuple would be: \((\text{JohnDoe}, \langle\text{Titanic,10}, \langle\text{Gladiator,9}, \langle\text{StarWars,8}\rangle\rangle)\). In other words, \( T \) represents the element combinations of dimensions from \( \text{recipient\_dim\_list} \) specified in the TO clause of the query. \( L_t \) consists of an ordered set of recommendations, each of which is represented by a tuple \( (r_{ij}, m_{ij}) \), where \( r_{ij} \in R \) and \( m_{ij} \in M \). Here \( R \) represents the element combinations of dimensions from \( \text{recommend\_dim\_list} \) specified in the RECOMMEND clause of the query (note that \( \text{recommend\_dim\_list} \) and \( \text{recipient\_dim\_list} \) must be mutually exclusive), and \( M \) represents the combinations of possible values for measures specified in \( \text{measure\_list} \) in the BY subclause. Specific tuples \( (r_{ij}, m_{ij}) \) are obtained from the processed ratings cube (i.e., after restrictions and aggregations specified in the query are done) by sorting all cells belonging to a given recipient \( t \) based on their measure values; these measure values \( m_{ij} \) and the corresponding element combinations of “RECOMMEND” dimensions \( r_{ij} \) constitute the contents of each recommendation in \( L_t \). \( L_t \) is further truncated according to the SHOW clause that limits the results to the top or bottom \( n \) recommendations.

If more than one measure is specified in \( \text{measure\_list} \), then the ordering is lexicographic. For example, in Query 8, Movies are first ordered based on the PersonalRating measure; if some records have the same value of PersonalRating, then those are further sorted based on PublicRating. Also, if the optional BY clause is not specified, the results are sorted according to the first measure in the BASED ON clause.

The output of a recommendation query can intuitively be represented as a matrix (cube) of the “TO” dimensions with the entries consisting of the lists of the elements representing the
“RECOMMEND” dimensions. In other words, one row in a recommendation matrix directly corresponds to one recommendation tuple described earlier. For example, Figure 3 shows the output matrices for two recommendations (movies to users and vice versa). The answer to the left query shows top two movies for each user, and the right one – the top two users for each movie (as specified in the SHOW clause). The output matrix produced for the left query in Figure 3 is based on users (as specified in the TO clause), and its cells contain movies (as specified in the RECOMMEND clause) and the corresponding rating measures (which were also used for sorting).

Note that, if the end-users want to use actual user names and movie titles in the output matrix, they should specify the output_attribute_list parameters from Figure 1 in the RECOMMEND and TO clauses. For example, the left recommendation in Figure 3 is stated as “RECOMMEND Movie(Title) TO User(Name)…” Also note that the recommendation results can be more complex in the sense that multiple dimensions can be used in the RECOMMEND and TO clauses, as Query 4 from Section 3.1 demonstrates. In such cases, each row in the output matrix can contain multiple dimensions and complex multidimensional entries in the cells.

Although we used the term “REQUEST queries” throughout the paper, recommendations are really not queries according to the standard meaning of the term, since they return a very idiosyncratic output of a recommendation matrix, which prevents the whole recommendation operation from being closed. To address this issue, we distinguish between the core REQUEST query containing RECOMMEND, TO, USING, RESTRICT, PREFILTER, POSTFILTER, and BASED ON clauses, and the recommendation wrapper containing RECOMMEND, TO, SHOW, and BY clauses (different aspects of RECOMMEND and TO are used in core and wrapper parts of the query). The core REQUEST query operates on a multidimensional cube of ratings and always returns the same type of an object – a cube of ratings. In contrast to this, the recommendation wrapper takes a multidimensional cube of ratings and transforms it to a different type of object –
the recommendation matrix that is subsequently returned as an output to the end-user. When
processing REQUEST queries, the core query is evaluated first, and then the wrapper is applied to
the output of the core REQUEST query.

![Figure 3. Generating recommendations from a multidimensional ratings cube.](image)

Although REQUEST is related to OLAP query languages, it has certain distinctive
characteristics pertaining to recommendations that make it different from other languages. In
particular, as explained above, REQUEST queries are divided into the “core” and “wrapper”
components, each component requiring separate evaluation methods. Also, ratings can be actual
(specified by the user) and inferred (from the actual ratings). Therefore, REQUEST provides
mechanisms for distinguishing between different types of ratings, as Query 7 demonstrated.
Finally, the language provides various other recommendation-specific properties, such as using a
single cube of ratings, the PRE- and POSTFILTER clauses, and recommendation-specific types of
aggregations, which make REQUEST uniquely suited for recommendation purposes.

### 3.3. Semantics of REQUEST Queries

Operational semantics of REQUEST is defined as the following sequence of operations over the

cube \textit{cube}_name from the USING clause of the query. Note that this operational semantics is only conceptual, i.e., the real query processing may be performed differently, but would result in the same exact outcome as explained below.

1. **Dimension restrictions.** First, the dimension restrictions specified in the RESTRICT clause, if present, produce a sub-cube of \textit{cube}_name by restricting some of its dimensions to include only a subset of their elements specified in the RESTRICT clause. For example, “RESTRICT User.Age \geq 18 AND Movie.Genre = \textquoteleft action\textquoteright ” produces a smaller cube having only the users with ages 18 and above and only the action movies. Section 3.2 lists the limits to the syntax of these restrictions (e.g., only comparisons of dimensional attributes to constants are allowed, no disjunctions in the RESTRICT clause). This amounts to applying restrictions in the RESTRICT clause one dimension at a time and also restricting one attribute at a time. The order in which these dimensions are restricted is unimportant since the final result does not depend on it.

2. **Measure-based cube filtering (before aggregation).** In this step, the cells of the restricted cube produced in Step 1 are further filtered based on measure restrictions specified in the PREFILTER clause. Since the measures supported by our multidimensional recommendation model can be either numeric or Boolean, the measure-based filtering capabilities of REQUEST include comparisons of both numeric and Boolean measures to specific values, as specified in Figure 1. Also note that this step filters \textit{individual} cube cells based on their measure values using multiple restrictions combined with AND and OR logical operators. Again, all measures mentioned in the PREFILTER clause have to be present in the schema of the \textit{cube}_name cube.

3. **Cube aggregation.** After performing restricting and prefiltering operations in Steps 1 and 2, the remaining cells of the obtained cube are aggregated according to the dimensions and their granularity levels specified in the mutually exclusive lists \textit{recommend\_dim\_list} and \textit{recipient\_dim\_list} from the RECOMMEND and TO clauses and according to the following three
rules: (i) if a dimension is specified in either RECOMMEND or TO clause by itself, i.e., without providing an aggregation attribute (e.g., Movie), then the cube is not aggregated along this dimension; (ii) if a dimension is specified in either RECOMMEND or TO clause with a corresponding aggregation attribute (e.g., Movie.Genre), then the cube is aggregated along this dimension based on the specified attribute (e.g., all individual movies are aggregated into their genres); (iii) if a dimension is omitted from both RECOMMEND and TO clauses, then the cube is aggregated fully along this dimension (i.e., in the resulting cube this dimension essentially disappears). Furthermore, the aggregation is done for the measures specified in the BASED ON clause. This clause also specifies the aggregation functions to be used for each measure. The currently supported numeric aggregation functions include AVG, SUM, MIN, and MAX, and the supported Boolean aggregation functions include CONJ (i.e., conjunction), DISJ (i.e., disjunction), and MAJORITY; however, the REQUEST language can be easily extended to support additional aggregation functions, such as AVG-in-TOP-k. If an aggregation function is not specified for the measures in the BASED ON clause, a default aggregation function for that measure is used (e.g., AVG). Essentially, this step represents a typical “roll-up” operation in OLAP systems. Figure 4 provides further illustration of the cube aggregation operation, where the Time dimension is collapsed and the User dimension is aggregated based on the gender attribute. Then all the movie ratings provided by users having a particular gender that were seen on different occasions (the shaded area) are averaged using the AVG function.

4. **Measure-based cube filtering (after aggregation).** In this step, the cells of the resulting aggregated cube can further be filtered based on the aggregated measure restrictions specified in the POSTFILTER clause. As it is specified in the BNF grammar in Figure 1, the syntax of the POSTFILTER clause is the same as for PREFILTER. Thus, if a given REQUEST query contains no aggregation, then PREFILTER and POSTFILTER, if both present, are combined into one
measure-based filtering operation. Also, all the measures mentioned in the POSTFILTER clause must appear among the measures mentioned in the BASED ON clause (because only these measures are aggregated). Note that the POSTFILTER clause is somewhat similar to the HAVING clause in SQL since both of them provide additional restrictions based on the aggregated data. However, one significant difference is that aggregation, including the HAVING clause, in SQL allows to create an arbitrary number of aggregated attributes from the same un-aggregated attribute, e.g., “SELECT MIN(Rating), MAX(Rating), SUM(Rating), COUNT(Rating), AVG(Rating) FROM Table”, whereas each measure can lead to just one aggregated version of itself in REQUEST. Another difference is that REQUEST supports not only numeric, but also Boolean aggregation functions (that are not available in standard SQL).

**Figure 4.** Cube aggregation operation.

5. **Generating recommendations.** In this step, the cube obtained in Step 4 is transformed into a specialized recommendation matrix, as was described in detail in Section 3.2. The rows of this matrix are determined by the TO clause of the REQUEST query. Each row of the matrix also contains the list of the records specified in the RECOMMEND clause and the measures are used to sort the results. These lists of records are sorted and truncated based on the SHOW clause.
This completes the description of semantics of the REQUEST language. This description was provided in a semi-formal manner in the sense that we did not use mathematics to define semantics of each of the 5 operations formally for the sake of readability. However, we provided enough details for the interested reader to easily understand and reconstruct formal semantic procedures defining each of these five steps. Moreover, we provide formal definition of the recommendation algebra in Section 4 which will make this reconstruction process even easier.

We next present a recommendation algebra that more formally defines how REQUEST queries are processed. Since algebraic operators should return objects of the same type as their inputs, we will target the recommendation algebra only to the core REQUEST queries (corresponding to Steps 1-4 above). To process a full REQUEST query, we construct an algebraic expression equivalent to the core REQUEST query, evaluate it, and then “feed” the results of the evaluation into the REQUEST wrapper to produce the final output.

4. Recommendation Algebra (RA)

Since multidimensional recommendations are based on the OLAP paradigm, we use the OLAP algebras introduced in the database community [1, 11, 14, 15, 22] to define the recommendation algebra. However, since the REQUEST language is tailored specifically for the domain of recommendations, only a subset of the standard OLAP operators is needed to process REQUEST queries. For example, we do not use a JOIN operator in the recommendation algebra because REQUEST works only on one cube, and we do not use PUSH and PULL operators because they do not occur in the recommendation contexts and can create problems as explained in Section 3.

In the rest of this section, we describe the recommendation algebra RA. We will follow the definitions of the OLAP operators introduced in [22] and assume that the ratings cube is defined as described in Section 2. The general syntax of recommendation algebra operators is:

\[ C_O = OP_{\text{parameters}}(C_i), \]
where $C_I = (D, M, H, E, L)$ denotes the input cube, $C_O = (D^*, M^*, H^*, E^*, L^*)$ the resulting output cube, $OP$ a recommendation algebra operator, and $parameters$ the parameters of operator $OP$.

We next introduce individual operators using this general syntax.

**Dimension restriction (DRSTR) operator.** This operator defines the “slice and dice” operation on the cube by putting restrictions on the dimensions. The simplest form of DRSTR operator is:

$$C_O = DRSTR_{P_{simple}}(C_I),$$

where $P_{simple}$ is a domain restriction based on a single dimension $d_i$, e.g., “User.Age > 21”. In other words, $P_{simple}$ is a Boolean function (a predicate) of the form $P_{simple}: E_i \to \{ \text{true, false} \}$. Given an arbitrary input ratings cube, as a result of this operator, only those cells that satisfy the given predicate are retained in the resulting cube. The calculation of $C_O$ is formally defined as:

- $D^* = D$, $M^* = M$, and $H^* = H$.
- $E_i^* = \{ e_i \in E_i \mid P_{simple}(e_i) \}$. Also, $E_j^* = E_j$, if $j \neq i$.
- $L^* = \{(address, content) \in L \mid address \in E_1^* \times \ldots \times E_n^* \}$.

In addition to the aforementioned simple predicates, this operator can also support more complex predicates. For example, $P_{complex}$ could be represented by a compound predicate of the form

$$P_{complex} = p_1 \ \text{AND} \ p_2 \ \text{AND} \ \ldots \ \text{AND} \ p_x$$

where each $p_j$ is a domain restriction involving a single dimension. Because the result of a simple predicate-based restriction is always a cube, the compound restriction operator can straightforwardly be implemented as a combination of simple restriction operators, i.e.,

$$C_O = DRSTR_{P_{complex}}(C_I) = DRSTR_{p_1 \ AND \ \ldots \ AND \ p_x}(C_I) = DRSTR_{p_1}(DRSTR_{p_2}(\ldots(DRSTR_{p_x}(C_I))\ldots)).$$

Note that, while DRSTR operator can support predicates with conjunctions (logical AND operations), it does not support arbitrary disjunctions (logical OR operations) because the result of such operations is no longer guaranteed to be a cube, as mentioned earlier. One scenario where disjunction $P$ of simple predicates can be supported is when all the predicates in the disjunction are restricting the same dimension. In this case, the resulting compound predicate is still of the
form \( P: E_i \rightarrow \{ \text{true}, \text{false} \} \), which, as mentioned earlier, always results in a cube.

**Measure restriction (MRSTR) operator.** This operator defines the “cell filtering” operation on the ratings cube by putting restrictions on measures. The simplest form of MRSTR operator is:

\[
C_0 = \text{MRSTR}_P(C_i),
\]

where \( P \) is a measure restriction based on a single measure \( m_j \). The restrictions can be based on a numeric measure (“PersonalRating > 7”) and on a Boolean measure (“Consumed = false”).

Unlike the DRSTR operator (which is a “slice and dice” operator), the MRSTR operator performs simple filtering of cube cells and, therefore, can support more complex predicates, e.g.,

\[
P = p_1 <\text{op}> p_2 <\text{op}> \ldots <\text{op}> p_x,
\]

where \(<\text{op}>\) represents a logical operator AND or OR. Given an arbitrary input cube, as a result of this operator, only the content of cells that satisfy the given predicate are retained in the resulting cube. The content of all other cells is assigned to NULL (these cells are retained and not deleted in order to maintain the proper cube structure).

The calculation of \( C_0 \) is formally defined as follows:

- \( D^* = D, M^* = M, H^* = H, \) and \( E^* = E \).
- Assign \( L^* = L \). Then, \( \forall (address, content) \in L^*: \) if \( \neg P(content) \) then \( L^*[address] = \text{NULL} \).

**Metric projection (MRPJ) operator.** This operator restricts the output of a ratings cube to include only a subset of the original set of measures. The simplest form of MRPJ operator is:

\[
C_0 = \text{MRPJ}_{m_j}(C_i),
\]

where \( m_j \) is a measure to be projected out. The calculation of \( C_0 \) is formally defined as follows:

- \( D^* = D, H^* = H, \) and \( E^* = E \).
- \( M^* = M \setminus \{ m_j \} \).
- Assign \( L^* = L \). Then, \( \forall (address, content) \in L^*: \) remove the \( j^{\text{th}} \) measure from \( L^*[address] \). If, as a result, \( L^*[address] \) has no more measures left, assign \( L^*[address] = \text{NULL} \).

If a set of metrics \( M' = \{ m'_1, \ldots, m'_x \} \) is needed to be removed at once, a more complex MRPJ operator can be implemented as a combination of simpler MRPJ statements. In other words,
\[ C_O = MRPJ_M'(C_I) = MRPJ_M'!\ldots(MRPJ_M'!(C_I))\ldots. \]

**Destroy dimension (DTDM) operator.** This operator reduces dimensions of the resulting ratings cube by including only a subset of the original set of dimensions. The simplest form of DTDM is
\[ C_O = DTDM_{d_i}(C_I), \]
where \(d_i\) is a dimension to be destroyed. Note that, if we destroyed dimension \(d_i\) by just removing its component from all cube cell addresses, we would have a number of cells in the cube with the same exact addresses, which leads to ambiguous results and, therefore, is undesirable. One way to deal with this situation is to aggregate all cells with the same address into a single cell in a resulting cube. Since, as we describe later, we already have an operator for cell aggregation (i.e., AGGR), we do not introduce the aggregation capability into DTDM. Therefore, in order to properly destroy dimension \(d_i\), we restrict the use of the DTDM operator only to the situations in which there is no ambiguity and no loss of information in the resulting cube. Thus, dimension \(d_i\) can be destroyed only when it has been maximally aggregated: \(|E_i| = 1\).

The calculation of \(C_O\) is formally defined as follows:

- If \(|E_i| > 1\), abort processing and return the same ratings cube, i.e., \(C_O = C_I\). Otherwise, continue as specified below.
- \(M^* = M, D^* = D - \{d_i\}, H^* = H - \{H_i\}, \) and \(E^* = E - \{E_i\}\).
- Assign \(L^* = L\). Then, \(\forall (address, content) \in L^*\): remove the \(i\)th dimension from \(address\).

If a set of dimensions \(D' = \{d'_1, \ldots, d'_s\}\) is needed to be removed at once (assuming they are all maximally aggregated), a more complex DTDM operator can be easily implemented as a combination of simpler DTDM statements. In other words, \(C_O = DTDM_{D'}(C_I) = DTDM_{d'_1}(\ldots(DTDM_{d'_s}(C_I))\ldots).\)

**Aggregation (AGGR) operator.** The aggregation operator performs aggregation on one or more dimensions and applies aggregation functions, such as SUM, AVG, etc., to each of the measures of the cube based on dimensions specified as grouping attributes. The general form of AGGR is:
Here $d_i.x_i$ represents a grouping attribute for dimension $d_i$ (specified only for dimensions that need to be grouped). Also, $m_j.f_j$ is an aggregation function specified for each cube measure $m_j$. If the aggregation function is not specified for some measure, a default aggregation function for that measure is used. After aggregation, $d_i.x_i$ becomes a dimension (with its own attributes, whichever appropriate) by replacing $d_i$. Our model also provides a special option to aggregate the dimension completely by specifying $d_i.x_i$ as $d_i.ALL$ (instead of using some attribute name). Furthermore, $m_j.f_j$ can be one of the standard numeric aggregation functions (e.g., MAX, MIN, AVG, SUM) or Boolean aggregation functions, including (the user can choose whichever is more appropriate):

$$f_j(B) = \text{DISJ}(B) = \bigvee_{b \in B} b, \quad f_j(B) = \text{CONJ}(B) = \bigwedge_{b \in B} b,$$

and

$$f_j(B) = \text{MAJORITY}(B) = \begin{cases} \text{true,} & \text{if } |\{ b \in B \mid b = \text{true} \}| \geq |\{ b \in B \mid b = \text{false} \}| \\ \text{false,} & \text{otherwise} \end{cases}.$$ 

Note that all three functions have some nice properties. E.g., if $(\forall b \in B) \ b = \text{true}$, then DISJ($B$) = CONJ($B$) = MAJORITY($B$) = true. Similarly, if $(\forall b \in B) \ b = \text{false}$, then DISJ($B$) = CONJ($B$) = MAJORITY($B$) = false. The three functions differ when input data $B$ contains mixed values.

As an example, consider the movie recommender system with three dimensions: User, Movie, and Time. Suppose user John Doe wants to know which movies that he has not seen yet are most relevant to him, regardless of when he is planning to watch them. Suppose the system has the following four ratings for John Doe: [(John Doe, Gladiator, weekday), (Rating=8, Consumed=true)], [(John Doe, Gladiator, weekend), (Rating=9, Consumed=false)], [(John Doe, Titanic, weekday), (Rating=7, Consumed=false)], [(John Doe, Titanic, weekend), (Rating=8, Consumed=false)]. In this case, the aggregation operator would look like:

$$C_O = \text{AGGR}_{(\text{Time}, \text{ALL}),(\text{Rating}, \text{AVG}, \text{Consumed}, \text{DISJ})}(C_1).$$

Since the time dimension has to be aggregated completely, we use AVG function to aggregate
the ratings for the same movie; we also use DISJ Boolean aggregation function to make sure that movies with at least one Consumed rating would not get recommended (since the user has seen it already). In this case, the results of aggregation would be: [(John Doe, Gladiator, ALL), (Rating=8.5, Consumed=true)], [(John Doe, Titanic, ALL), (Rating=7.5, Consumed=false)], and the user, by filtering on the “Consumed=false” status flag, would be able to receive a correct recommendation of “Titanic”, since the user has already seen “Gladiator”.

Now consider the same four ratings, but a different scenario, where John Doe wants to know which times of week seem to be best for him in terms of movie watching, regardless of what kind of movie he is planning on watching. In this case, the aggregation operator would be:

\[ C_O = \text{AGGR}_{\text{Movie}, \text{ALL}, \text{Time}, \text{TimeOfWeek}}(\text{Rating, AVG, Consumed, CONJ})(C_I). \]

Since the movie dimension has to be aggregated completely, we again use AVG function to aggregate the ratings for the same time values. However, this time it may make more sense to use the CONJ Boolean aggregation function to make sure that only time periods with no unseen movies (i.e., with no Consumed=false flags) would get labeled as Consumed=true (since only in that case there would not be anything for the user to watch during that time period). Note that, while in the current model we use CONJ, DISJ, and MAJORITY functions, other Boolean aggregation functions are also possible.

The calculation of \( C_O \) is then formally defined as follows:

- \( D^* = D - \bigcup_i \{ d_i \} + \bigcup_i \{ d_i.x_i \} \). Note that, if \( d_i.x_i = \text{Root}(H_i) \) then dimension \( d_i \) remains unchanged, i.e., there is no aggregation on \( d_i \). As a result, \( E_i \) and \( H_i \) (see below) would also remain unchanged.
- \( M^* = M \).
- \( \forall i = 1, \ldots, n: H_i^* = \text{SubGraph}(H_i, d_i.x_i) \). In other words, the attributes for the newly aggregated dimension are the ones that are uniquely determined by the new key attribute \( d_i.x_i \) (i.e., that are reachable from \( d_i.x_i \) in the attribute hierarchy for dimension \( d_i \)). Furthermore, after aggregation, only the hierarchy structure rooted in node \( d_i.x_i \) is needed for further processing. For example, based on Time dimension attribute hierarchy (i.e., Time → DayOfWeek → TimeOfWeek), after aggregating Time dimension based on DayOfWeek the new set of attributes would be \{DayOfWeek, TimeOfWeek\}.  


• \( \forall i = 1, \ldots, n: \quad E_i^* = \text{dom}(d_i, x_i) \). The cube cells along the newly aggregated dimension become labelled with the values of the new key attribute. For example, after aggregating Time dimension based on DayOfWeek, the cube cells for this dimension would be labeled as \{ Mon, Tue, Wed, Thu, Fri, Sat, Sun \}.

• \( \left( \forall address^* \in E_1^* \times \cdots \times E_n^* \right) \left( \forall j = 1 \ldots k \right) \quad L'[address^*]m_j = \text{aggr}_{f_j} \left( L[address]m_j \right) \). In other words, each metric \( m_j \) is computed for each cell of the new cube using aggregation function \( f_j \) and based on cells from input cube that were replaced by (or aggregated into) a given cell. More precisely, given \( address = (e_1, \ldots, e_n) \in E_1 \times \cdots \times E_n \) and \( address^* = (e_1^*, \ldots, e_n^*) \in E_1^* \times \cdots \times E_n^* \), we say that \( address \prec address^* \) if and only if \( d_i = e_i \Rightarrow d_i x_i = e_i^* \). Finally, note that cube cells that have NULL values are ignored during the aggregation. However, if all underlying cells have NULL values for a specific aggregation, then that aggregated cell will be assigned the NULL value as well.

The recommendation algebra RA is formed by the composition of these five operators. Since each of these operators takes a rating cube and produces another rating cube, the RA algebra is closed. For example, Query 3 (recommend top 5 movies to the user to see over the weekend, but only if the personal ratings of the movies are higher than 7) can be expressed in RA as:

\[
\text{MRSTR}_{\text{PersonalRating} > 7} \left( \text{DTDM}_{\text{Theater, Time, Companion}} \right) \left( \text{AGGR}_{\text{Theater.ALL, Time.ALL, Companion.ALL}} \left( \text{MRPJ}_{\text{PublicRating, Consumed}} \left( \text{DRSTR}_{\text{Time.WeekTime} = \text{"Weekend"}} \left( \text{MovieRecommender} \right) \right) \right) \right)
\]

As explained before, this algebraic expression specifies only the core part of the REQUEST query. The actual recommendation results are generated by the REQUEST wrapper from the results of core query. Therefore, this algebraic expression destroys all other dimensions towards the end, leaving only the User and Movie dimensions for the wrapper to work with. Also, this example shows how MRPJ and DTDM operators remove measures and dimensions from the cube; e.g., PublicRating and Consumed measures as well as Theater, Time, and Companion dimensions are removed from the MovieRecommender cube.

5. Mapping REQUEST Queries into Recommendation Algebra RA

The translation of the “core” part of the REQUEST query to the recommendation algebra is fairly straightforward, because the REQUEST language was developed based on the underlying algebra.
In particular, the mapping is performed by parsing the query and generating corresponding algebraic operators. The MAP algorithm, presented in Figure 5, shows how to translate an arbitrary Core-REQUEST query with its specific parameters, such as various aggregations as well as measure or dimension restrictions, into an algebraic expression in RA.

As mentioned earlier, the most general form of the REQUEST query is:

\[
REQUEST\_query ::= \text{RECOMMEND } \text{recommend\_dim\_list} \text{ TO recipient\_dim\_list} \\
\text{USING cube\_name} \\
[ \text{RESTRICT dimension\_restrictions} ] \\
[ \text{PREFILTER preaggregation\_measure\_restrictions} ] \\
\text{BASED ON aggr\_measure\_list} \\
[ \text{POSTFILTER postaggregation\_measure\_restrictions} ] \\
[ \text{SHOW measure\_rank\_restriction} ]
\]

Based on the input query \textit{REQUEST\_Query}, the MAP algorithm produces a corresponding algebraic expression \textit{RA\_op} in RA. By default, initially \textit{RA\_op} is assigned the identity operator \textit{ID} (Line 1), i.e., \textit{ID}(\text{cube}) \equiv \text{cube} for any \text{cube} instance. First, the MAP algorithm checks whether \textit{REQUEST\_Query} has any restrictions on dimensions (Line 2) and if so, MAP then generates a dimension restriction operator \textit{DRSTR} with corresponding parameters (Line 3). Note, that the MAP algorithm continuously builds the resulting algebraic expression \textit{RA\_op} by composing the current value of \textit{RA\_op} with each newly generated operator. For notational purposes, we use the \(\oplus\) symbol to represent the composition of two algebraic operators, i.e., \(op_1 \oplus op_2 (\text{cube}) = op_2(op_1(\text{cube}))\) for any \text{cube} and any algebraic operators \textit{op}_1, \textit{op}_2. After dimension restrictions, MAP checks whether \textit{REQUEST\_Query} has any restrictions on measures (Line 4) and if so, it then generates a measure restriction operator \textit{MRSTR} with corresponding parameters (Line 5). Once dimension and measure restrictions are applied, the aggregation is performed next. The measures to be aggregated and their aggregation functions are specified by the user in the \text{BASED ON} clause of the query, but first the unused measures (measures that do not appear in this clause) are projected out using operator \textit{MRPJ} (Lines 6-7).
MAP(REQUEST_query) {
  (1) RA_op := ID
  (2) if (∃ RESTRICT clause in REQUEST_query) then
      RA_op := RA_op ⊕ DRSTR(dimension_restrictions)
  (3) if (∃ PREFILTER clause in REQUEST_query) then
      RA_op := RA_op ⊕ MRSTR(preaggregation_measure_restrictions)
  (4) foreach m ≠ aggr_measure_list in BASED ON clause
      RA_op := RA_op ⊕ MRPJ_m
  (5) dimension_aggregations = ∅
  (6) foreach d_i ∈ cube_name
      if d_i ∉ recommend_dim_list ∪ recipient_dim_list then
          dimension_aggregations := dimension_aggregations ∪ { d_i.ALL }
      else if (∃ x) d_i.x ∈ recommend_dim_list ∪ recipient_dim_list then
          dimension_aggregations := dimension_aggregations ∪ { d_i.x }
      measure_aggregations = ∅
  (7) foreach (m_j, aggr_j) ∈ aggr_measure_list in BASED ON clause
      measure_aggregations := measure_aggregations ∪ { m_j.aggr_j }
  (8) RA_op := RA_op ⊕ AGGR(dimension_aggregations), measure_aggregations)
  (9) foreach d ≠ recommend_dim_list ∪ recipient_dim_list
      RA_op := RA_op ⊕ DTDM_d
  (10) if (∃ POSTFILTER clause in REQUEST_query) then
      RA_op := RA_op ⊕ MRSTR(postaggregation_measure_restrictions)
  (11) return RA_op;
}

Figure 5. Mapping Core-REQUEST queries into RA expressions.

Subsequently, operator AGGR is generated (Line 17) with parameters dimension_aggregations and measure_aggregations, where the former specifies the granularity (or aggregation) levels for all dimensions that need to be grouped (Lines 8-13) and the latter specifies aggregation functions for all measures (Lines 14-16). Subsequently, all irrelevant (and fully aggregated) dimensions, i.e., the dimensions that do not appear in RECOMMEND and TO clauses, are destroyed using operator DTDM (Lines 18-19). Finally, MAP checks whether REQUEST_Query has any post-aggregation restrictions on measures (Line 20) and if so, it then generates a measure restriction operator MRSTR with corresponding parameters (Line 21). Finally, Line 22 returns the resulting algebraic expression RA_op, and the query results can be obtained by applying RA_op to the input cube cube_name specified in the USING clause.

We next explore a formal relationship between the Core-REQUEST queries and RA. To do
this, we first introduce some preliminary concepts. Let \( o \) be a specific instance of any of the five RA operators, for example, \( o_i = \text{DRSTR}_{(\text{Movie.Genre} = \text{“comedy”})} \). Given recommendation cube \( C \), we say that \( o \) is a well-defined operation for \( C \) if \( o(C) \) can be successfully performed based on the schema of cube \( C \) as well as the dimensions, attributes, and measures specified in operator \( o \). For example, operator \( \text{DRSTR}_{(\text{Movie.Genre} = \text{“comedy”})} \) is well-defined for any cube that has dimension \( \text{Movie} \) with an attribute \( \text{Genre} \) and is not well-defined for any other cube.

The notion of the well-defined operation can be directly extended from a single algebraic operator to sequences of operators. Let \( s \) be a sequence of RA operators, i.e., \( s = <o_1, \ldots, o_n> \), where each \( o_i \) is a specific instance of any of the five RA operators. We say that \( s \) is a well-defined operation sequence for recommendation cube \( C \) if operation \( o_n(o_{n-1}(\ldots(o_2(o_1(C)))))) \) can be successfully performed in the sense that each operator \( o_i \) in the sequence is well-defined for its input cube: operator \( o_1 \) is well-defined for cube \( C \), \( o_2 \) is well-defined for cube \( o_1(C) \), etc., i.e., \( o_i \) is well-defined for cube \( o_{i-1}(\ldots(o_1(C))) \) for each \( i = 2, \ldots, n \).

**Lemma 1** [safe swap of DRSTR forward]. Let \( o_{\text{DRSTR}} \) be an instance of the DRTSR operator, \( o_{\text{ANY}} \) be an instance of any of the five recommendation algebra operators (i.e., DRSTR, MRSTR, MRJP, DTDM, AGGR), and \( C \) be a recommendation cube where \( o_{\text{DRSTR}}(o_{\text{ANY}}(C)) \) is well-defined. Then, \( o_{\text{ANY}}(o_{\text{DRSTR}}(C)) \) is also well-defined and \( o_{\text{DRSTR}}(o_{\text{ANY}}(C)) = o_{\text{ANY}}(o_{\text{DRSTR}}(C)) \).

**Proof.** Immediate from the definitions of RA operators. \( \square \)

**Lemma 2** [safe swap of DTDM back]. Let \( o_{\text{DTDM}} \) be an instance of the DTDM operator, \( o_{\text{ANY}} \) be an instance of any of the five RA operators, and \( C \) be a recommendation cube where \( o_{\text{ANY}}(o_{\text{DTDM}}(C)) \) is well-defined. Then, \( o_{\text{DTDM}}(o_{\text{ANY}}(C)) \) is also well-defined and \( o_{\text{ANY}}(o_{\text{DTDM}}(C)) = o_{\text{DTDM}}(o_{\text{ANY}}(C)) \).

**Proof.** Immediate from the definitions of RA operators. \( \square \)
Lemma 3 [safe swap of MRPJ back]. Let $o_{\text{MRPJ}}$ be an instance of the MRPJ operator, $o_{\text{ANY}}$ be an instance of any of the five RA operators, and $C$ be a recommendation cube where $o_{\text{ANY}}(o_{\text{MRPJ}}(C))$ is well-defined. Then, $o_{\text{MRPJ}}(o_{\text{ANY}}(C))$ is also well-defined and $o_{\text{ANY}}(o_{\text{MRPJ}}(C)) = o_{\text{MRPJ}}(o_{\text{ANY}}(C))$.

Proof. Immediate from the definitions of RA operators.

Theorem 1 [canonical form of recommendation algebra]. Let $C$ be a recommendation cube. Then, for any sequence $s$ of recommendation algebra operators, such that $s(C)$ is a well-defined operation, there exists a corresponding canonical sequence $s'$ of the form:

$$s' = <[\text{DRSTR}],[\text{MRSTR}],[\text{AGGR},[\text{MRSTR}]]^*,[\text{DTDM}],[\text{MRPJ}]>$$

that is equivalent to $s(C)$, i.e., $s'(C) = s(C)$, and where the number of AGGR operators in $s'$ is equal to the number of AGGR operators in $s$.

Proof. The proof is provided in the Appendix.

We next establish the relationship between Core-REQUEST and RA.

Theorem 2. RA is strictly more expressive than Core-REQUEST.

Proof. Obviously, RA is at least as expressive as the Core-REQUEST language, because every Core-REQUEST statement can be expressed in RA using the MAP algorithm. Furthermore, directly from the MAP algorithm we have that all Core-REQUEST queries are of the following algebraic form: $<[\text{DRSTR}],[\text{MRSTR}],[\text{MRPJ}],[\text{AGGR}],[\text{DTDM}],[\text{MRSTR}]>$. Based on Lemmas 2 and 3 and also on the fact that, in the absence of AGGR operator, PREFILTER and POSTFILTER can be represented as one measure restriction operation (as mentioned in Section 3), all Core-REQUEST queries can also be expressed by the following equivalent sequence: $<[\text{DRSTR}],[\text{MRSTR}],[\text{AGGR},[\text{MRSTR}]],[\text{DTDM}],[\text{MRPJ}]>$. Based on Theorem 1, it is clear that RA can produce the expressions of a strictly more general form, i.e., $<[\text{DRSTR}],[\text{MRSTR}],[\text{AGGR},[\text{MRSTR}]]^*,[\text{DTDM}],[\text{MRPJ}]>$, where the precise difference in expressive

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2 Using traditional notation, the square brackets denote that a particular operator is optional, and the star symbol (*) in $(\text{AGGR},[\text{MRSTR}])^*$ denotes zero, one, or more repetitions of AGGR and, possibly, MRSTR operators.
power lies in the RA’s ability to specify multiple \(<\text{AGGR}, [\text{MRSTR}]>\) operator sequences (as opposed to 0 or 1 such sequences in Core-REQUEST).

Theorems 1 and 2 explain the differences between expressive powers of RA and Core-REQUEST at the theoretical level: Core-REQUEST allows at most one aggregation operation in a query, while RA supports multiple aggregations because the algebra is closed. For example, in a 2-dimensional recommendation application with User and Movie dimensions and one measure, Rating, the following RA expression

\[
\text{AGGR}_{\text{User.ALL, Movie.ALL}}(\text{Rating.AVG})(\text{MRSTR}(\text{Rating} > 7)(\text{AGGR}_{\text{User.Profession, Movie.Genre}}(\text{Rating.AVG})(C)))
\]

cannot be expressed in Core-REQUEST.

One way to address the issue that Core-REQUEST is strictly less expressive than RA is to extend REQUEST in such a way that their expressive powers would become equal. According to Theorems 1 and 2, this would mean providing support for multiple aggregations (and optional measure restriction capabilities after each aggregation) in REQUEST. However, we decided against this because such multiple aggregations (a) do not occur naturally in recommendation applications and, therefore, have a very limited need in the real-world applications, and (b) unnecessarily complicate the language by adding extra complexity and make it less user-friendly without providing any tangible benefits from these extensions.

6. Conclusions
In this paper we introduced language REQUEST for specifying user-driven recommendations. REQUEST queries are formulated on multidimensional cubes of ratings, support OLAP-based aggregation capabilities, are expressed in a simple declarative language capturing idiosyncrasies of recommender systems and thus provide the several advantages to the users of recommender systems. In particular, REQUEST empowers the end-users by letting them customize recommendations by formulating them in the ways that satisfy their individual needs in a flexible
and user-friendly manner. Also, unlike SQL, which constitutes a general-purpose database query language, REQUEST is designed specifically for multidimensional recommender systems. Therefore, its constructs are developed exclusively for specific recommendation contexts, and every REQUEST query can be directly interpreted as a recommendation. As a result, REQUEST can express complex recommendations in a concise and clear manner. Finally, REQUEST design follows the multidimensional data model and does not depend on any of its particular implementations (one can use ROLAP, MOLAP or hybrid OLAP approaches to implement multidimensional recommender systems).

We also presented an OLAP-based recommendation algebra, showed how REQUEST recommendations can be expressed in it, and provided in-depth analysis of its expressiveness. Therefore, REQUEST queries can be processed using this mapping similarly to how SQL queries are processed in relational DBMSes. One query processing problem pertaining to recommender systems deals with the determination of which new ratings need to be evaluated in order to answer a particular REQUEST query, assuming that the entire cube of ratings cannot be pre-computed ahead of time. For example, in order to answer the query “which movies to recommend to Jane Doe to see on March 5 on Saturday night with her boyfriend in a movie theatre,” the system may not need to estimate all the ratings on-the-fly in the recommendation cube described in Example 1. Since rating estimation becomes query-dependent, an interesting and challenging problem is to determine the subset of ratings that needs to be estimated to answer a given query. We plan to study this problem in the future.

Finally, it is important to develop a good GUI-based front-end to REQUEST so that naïve end-users would be able to express their user-driven recommendations using this interface. Designing such interface constitutes another topic of our future research.
References


**Theorem 1** [canonical form of recommendation algebra]. Let $C$ be a recommendation cube. Then, for any sequence $s$ of recommendation algebra operators, such that $s(C)$ is a well-defined operation, there exists a corresponding canonical sequence $s'$ of the form:

$$s' = <[\text{DRSTR}], [\text{MRSTR}], (\text{AGGR}, [\text{MRSTR}])^*, [\text{DTDM}], [\text{MRPJ}] >$$

that is equivalent to $s(C)$, i.e., $s'(C) = s(C)$, and where the number of AGGR operators in $s'$ is equal to the number of AGGR operators in $s$.

**Proof.** The proof is by construction and follows from Lemmas 1-3. First, let $s' := s$. Based on Lemma 1, all DRSTR operators (if any present in sequence $s$) can be moved to the front of the sequence by repeatedly swapping these operators with preceding ones, without affecting the result of the overall algebraic operation. Similarly, based on Lemmas 2 and 3, all DTDM and MRPJ operators (if any) can be safely moved to the end of the sequence. Furthermore, if there are several DRSTR operators in $s$, after being moved to the beginning of the sequence, they can all be combined into a single DRSTR operator (as discussed in Section 4): $\text{DRSTR}_{p1}(\text{DRSTR}_{p2}(\ldots (\text{DRSTR}_{px}(C)\ldots ))) = \text{DRSTR}_{p1\ AND\ ...\ AND\ px}(C)$. Analogous statements can also be made for the multiple DTDM as well as multiple MRPJ operators (as discussed in Section 4). And if there are no DRSTR (or DTDM, or MRPJ) operators in $s$, they will be omitted in $s'$ as well. Thus, at this point we have that $s'$ is of the form: $<[\text{DRSTR}], s'_{\text{AGGR,MRTSR}}, [\text{DTDM}], [\text{MRPJ}]>$, where $s'_{\text{AGGR,MRTSR}}$ is the sub-sequence of only AGGR and MRSTR operators present in $s$ (in their original order).

Because generally one cannot safely swap two instances of AGGR operator (as illustrated in Figure A1), the exact sequencing of AGGR operator instances in $s$ must be preserved in $s'$. Thus, $s'$ will have the same exact AGGR operators as in $s$ (and in the same sequence). Furthermore, because generally one cannot safely swap instances of AGGR and MRSTR operators (as
illustrated in Figure A2), the relative sequencing between AGGR and MRSTR also is preserved in $s'$. As a result, $s_{AGGR,MRSTR}$ can always be expressed with the following equivalent pattern: $<[MRSTR],[AGGR,[MRSTR]]^*>$, since adjacent MRSTR operators (if any present) can always be combined into one MRSTR operator.

We have constructed sequence $s'$ of the form $<[DRSTR],[MRSTR],[AGGR,[MRSTR]]^*,[DTDM],[MRPJ]>$ only by safely swapping adjacent operators. Thus, $s'(C) = s(C)$, and the number of AGGR operators in $s'$ is equal to the number of AGGR operators in $s$.

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**Figure A1.** Non-safe swapping of two AGGR instances.

<table>
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<th>Movies $\Rightarrow$ Users</th>
<th>AGGR$_U$</th>
<th>Movies $\Rightarrow$ Users</th>
<th>AGGR$_M$</th>
</tr>
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</tr>
<tr>
<td>Y 7</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Abbreviations:**

AGGR$_U$: $AGGR_{User\,ALL}(\text{Rating\,AVG})$

AGGR$_M$: $AGGR_{Movie\,ALL}(\text{Rating\,AVG})$

Not equal

---

**Figure A2.** Non-safe swapping of the AGGR and MRSTR instances.

<table>
<thead>
<tr>
<th>Movies $\Rightarrow$ Users</th>
<th>AGGR</th>
<th>Movies $\Rightarrow$ Users</th>
<th>MRSTR</th>
</tr>
</thead>
<tbody>
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<td></td>
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<td></td>
</tr>
<tr>
<td>X 1 9</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Y 7 3</td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

**Abbreviations:**

AGGR: $AGGR_{User\,ALL}(\text{Rating\,AVG})$

MRSTR: $MRSTR_{\text{Rating}>2}$

Not equal

---

Abbreviations:

AGGR: $AGGR_{User\,ALL}(\text{Rating\,AVG})$

MRSTR: $MRSTR_{\text{Rating}>2}$