Supply Contracts with Options in E-Business

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Abstract

In this paper we devise a generic framework to value supply contracts in the presence of a spot market. More specifically, we consider an $N$-period setting where a manufacturer can, after paying an up-front reservation price, reserve capacity with her long-term supplier via a long-term contract with options knowing that she can also access a spot market to meet her procurement needs or to build up inventory for future demands. We show that a purchase-up-to policy is optimal and determine the optimal option reservation capacity level even when the demand and spot price distributions are non-stationary and correlated. Furthermore, we define the value of supply flexibility as the difference between the expected values of a supply contract with options and a supply contract with fixed quantity commitment per period, and we derive the value of spot price information to the manufacturer. Furthermore, we show that in the presence of a spot market the value of any supply contract, which can be represented by a combination of a contract with fixed quantity commitment and a supply options contract, is independent of the inventory problem, i.e., the only use of supply contracts is to hedge against spot price uncertainty.

Keywords: Supply Contracts; Spot Market; Real Options; Value of Information; Value of Quantity Flexibility
1 Introduction & Motivation

Supply chain management aims to optimize the management of material, information, and financial flows in a network of suppliers and buyers to efficiently respond to market demands for goods and services. It spans the full supply chain from the procurement of raw materials to the delivery of finished goods to the ultimate consumers and therefore involves a variety of issues including among many others, product/process design, supply planning, production and inventory planning as well as inbound/outbound logistics.

The advent of the internet facilitated many of the supply chain-related processes by allowing the automation of processes and thus reducing transaction costs and increasing information and financial flows. The internet also brought into existence on-line trading communities (so called spot markets or internet-based exchanges), where suppliers can sell off excess inventory and buyers can fill last minute procurement needs.

Our focus in this paper is on supply and procurement planning in general and on contracts between suppliers and buyers in the presence of such a spot market, in particular. Almost all business-to-business transactions are governed by contracts and as a consequence the academic literature on supply contracts is quite rich. In the recent past, supply contracts with options, which give the buyer the right to purchase a specified amount of a good or service without the obligation to do so, have gained more and more momentum in many industries, mainly in industries in which there are also spot markets, such as in the utilities, the textile, and the electronics and high-tech industry.

This is not surprising as options and other derivatives are a main tool in managing the risk associated with the uncertainty due to fluctuating prices, availability, and demand. However, only a few academics (e.g., Barnes-Schuster et al. (2000), Ritchken et al. (1986), and Wu et al. (2001a and b) consider supply contracts with options and
even fewer allow for spot markets as an additional procurement channel for the buyer (e.g., the excellent papers by Wu et al. (2001a and b)). The papers that are closest to ours are the ones by Wu et al. (2001a and b). Wu et al. model contracting contractual interactions between one or more buyers and one or more sellers in capital intensive industries or in industries where scaling capacity requires long lead times. The goal of their papers are to analyze the trade-off between long-term contracting and spot market purchases considering a contract structure similar to our options contract, i.e., specifying an up-front reservation cost per unit of capacity and an execution/exercise cost per unit of output when the reserved capacity is actually used. The contribution of our paper is that in addition to the options and spot market they consider we consider storable items and account for the role of inventory, which is a significant step towards capturing real-world needs.

The objective of our work is to illustrate the benefit of options contracts as a risk management tool for hedging against price and demand uncertainty in inventory procurement decisions. We also emphasize the flexibility that is provided by option contracts in operations and procurement applications. More specifically, we determine the optimal purchasing policy for a buyer (which we will refer to as the manufacturer) in an $N$-period time horizon who has access to both supply contracts with options and a spot market for the procurement of components given that she also carries inventory as a buffer against demand and spot price uncertainty. Allowing for non-stationary demand and spot price distributions, which may be correlated with each other or from period to period, we derive the value of the quantity flexibility provided by the supply contract with options to the manufacturer. Our analysis leads us to the “fair market” price for supply contracts with options if the spot price distributions are independent of the manufacturer’s demand.

To complete our study, we consider a modified sequence of events and include the actual spot price realization in the available information to the manufacturer when she
is making her decision to exercise her options. This allows us to determine the value of spot market information to the manufacturer.

We extend the manufacturer’s analysis in a companion paper (Kleinknecht et al. 2002) to a Stackelberg game between the manufacturer and her supplier with the supplier acting as the leader in anticipation of the manufacturer’s response.

2 Overview of the Paper

We start our analysis with the study of a risk-neutral manufacturer who can purchase a supply options contract that allows her to procure up to $K$ units in every period $n$, $n \in \{1, \ldots, N\}$, for a unit price of $p_n(K)$ over a time horizon comprised of $N$ periods. In order to enter into this supply agreement with her supplier, the manufacturer has to pay an up-front reservation price of $R(K, N)$. In addition, the manufacturer can also replenish her inventory from a spot market that we assume to be liquid, i.e., the manufacturer can buy as many units as she needs via the spot market at the prevailing unit price, $\pi_n$. In the first part of the paper, the timing is such that the manufacturer has to make the exercise decision involving the actual number of units to buy in $[0, K]$ before she knows the actual spot price realization in that period, i.e., based only on a conditional expectation given the most recent spot price and demand, which corresponds to industry practice in the energy market. After the spot price is announced, the manufacturer can buy additional units or even sell off part of her inventory on the spot market. The final step in each period is that demand is realized and holding and shortage costs occur.

We show that the problem admits a modified newsboy solution and determine the optimal purchase-up-to policy in every period, which is independent of the number of options exercised. Thus, we find that the supply options contract between the manufacturer and her supplier is purely a financial derivative on the spot market,
i.e., independent of the manufacturer’s inventory problem, and serves only as a hedge against spot price uncertainty. Our analysis allows us to determine the value of the supply options contract and the value of supply flexibility, which we define as the difference in value between a supply contract with options and a supply contract where the manufacturer is obligated to order $K$ units in every period.

In Section 6, we generalize and modify the sequence of events and assume that the spot price is known to the manufacturer at the time of her exercise decision. We can thus calculate the value of spot price information to the manufacturer. Moreover, we determine the difference in the value of flexibility to the manufacturer between the two sequences of events. We conclude our paper with a summary of the results and a list of topics that we deem worthy of further research.

3 Framework & Assumptions

Suppose the planning horizon is $N$ periods and that we number the periods backwards, i.e., $N, N-1, \ldots, 1$. All our results are subject to the following assumptions and definitions:

3.1 Long-term Supply Contract

The contractual arrangement between the manufacturer and the supplier is defined as follows:

- In paying a reservation price, $R(K,N)$, the manufacturer has the right (but not the obligation) to order up to a maximum of $K$ units of supply from the long-term supplier in every period $n$, $n \in \{1,\ldots,N\}$, over the $N$-period time horizon. The associated exercise price per unit of supply ordered from the long-term supplier in
period \( n \) is \( p_n(K) \). \( R(K,N) \) is assumed to be a non-decreasing function of \( K \) and of \( N \) with \( R(0,N) = 0 \). With regard to the exercise prices, \( p_n(K), n \in \{1,\ldots,N\} \), (which are also referred to as the strike prices in the financial options literature) we only assume that they are non-negative and a function of \( K \), the number of units reserved. Furthermore, we consider that \( p_n(K) \leq \alpha p_{n-1}(K) + s_n \) and \( p_n(K) + h_n \geq \alpha p_{n-1}(K) \) for all \( n \in \{2,\ldots,N\} \), meaning that it is not worth while to wait for one period and willingly incur a shortage in the interim or to purchase via the contract in a period and then hold the items until the next period. It is easy to verify that setting the pricing parameters of the contract such that the last assumption is fulfilled is in most situations in the best interest of the supplier since unit holding costs are in many situations similar for both the supplier and the manufacturer.

**Remark 1** Notice that the long-term supply contract under consideration consists of a series of \( N \) equal European call options with one call option for \( K \) units of supply per period.

**Remark 2** Notice also that we could easily generalize our results to the case where the manufacturer acquires an option specifying a different amount in every period, i.e., \( K_n \) units for period \( n \) and \( K = (K_N,\ldots,K_1) \), implying that the unit exercise price in period \( n \) is \( p_n(K_n) \).

Supply contracts governing the specificities of buyer-seller transactions are widely used in industry. The specific type of options contract that we consider in this paper has found application among others in the utility (e.g., Worth Magazine 2001 and The Economist 2001), electronics, and semiconductor industries (see, for example, Semiconductor 1998 and Taiwan Economic 2000).
3.2 Spot Market

Throughout this paper, we consider a spot market with the following characteristics:

- There is a random, non-negative spot price $\pi_n$, which may be non-stationary, correlated from one period to the next, and correlated with the demand in each period $n, n \in \{1, .., N\}$. Notice that all spot prices are known in retrospect, i.e., in period $n$ all $\pi_m$ with $m > n$ are known. To reduce notational complexity, we assume that the spot prices are one-step Markovian, i.e., the distributions in period $n$ are entirely determined by the realization in period $n + 1$. However, all results apply to more general spot price distributions, which are correlated between more than just two subsequent periods, too.

- We assume that the spot market is sufficiently liquid in the sense that the manufacturer can buy as much as she wants (within her needs) or sell off her entire on-hand inventory without affecting the spot price, i.e., the spot price is independent of the manufacturer’s selling/purchasing quantity on the spot market.

- We also make a “no arbitrage” assumption in the following sense: It is not cheaper in expectation to buy in the spot market today and pay inventory holding cost for one period than to purchase in the spot market tomorrow, i.e.,

  \[
  \Pr\left(\pi_n + h_n \geq \alpha E_{d_n|d_{n+1}} \left[ E_{\pi_{n-1}|\pi_n} \left[ \pi_{n-1} \right] \right] \right) = 1\ 
  \]

  for all $n > 1$ where $\alpha$ denotes the period-to-period discount factor, $h_n$ is the one-period holding cost in period $n$, $\pi_n$ the spot price realization in period $n$, and $E_{d_n|d_{n+1}} \left[ E_{\pi_{n-1}|\pi_n} \left[ \pi_{n-1} \right] \right]$ the expected spot price in period $n – 1$ seen from period $n$.

**Remark 3** Although we do not make any specific assumptions regarding the spot price and demand distributions, other than the non-negativity and one-step Markovian property, we assume throughout this paper that all expectations exist, i.e., are finite.
Remark 4 Since this paper considers a finite-horizon stochastic dynamic program, the one-step Markovian assumption could be easily relaxed and the state space augmented without impacting our results.

3.3 Manufacturer

We make the following assumptions regarding the manufacturer:

- There is a finite number, $N$, of discrete periods in which non-negative demands, denoted by $d_n$ and with distribution function $F_{d_n}$ for $n \in \{1, \ldots, N\}$, occur. Demands may be non-stationary, correlated between subsequent periods, and correlated with the spot price. To simplify notations, we assume that the demands are one-step Markovian, i.e., the distributions in the current period, $n$, are entirely determined by the realization in the previous period, $n+1$. However, all results hold true even if the distributions are correlated between more than just two subsequent periods.

Remark 5 Again, since we deal with a finite-horizon stochastic dynamic program we could easily relax the assumption regarding the one-step Markovian nature of the demand process and generalize to any correlation between periods by just augmenting our state-space.

- Costs may or may not be discounted from period to period using the discount factor $\alpha$, where $0 < \alpha \leq 1$. Note that we could easily extend the analysis to the case where $\alpha_n \neq \alpha_m$ for all $n \neq m$. However, for the sake of simplicity of exposition, we consider that $\alpha_n = \alpha$ for all $n$.

- The holding and shortage costs are linear, with the unit holding cost in period $n$ being denoted by $h_n$ and the unit shortage cost by $s_n$, with $h_n, s_n \geq 0$. (Both of
them are expressed in terms of monetary units at the beginning of the period, so we do not have to discount them.)

- At the end of the last period, each unit of leftover inventory is salvaged at the prevailing spot price (if there is backlogged demand then the entire outstanding demand has to be met from the spot market). Otherwise, unsatisfied demand is fully backlogged from period to period (no lost sales) and leftover stock is carried over to the next period. However, we assume that demand can only be backlogged for one period, i.e., it has to be fully met at least one period later.

- The objective is to minimize the expectation of the costs to be incurred over the N-period time horizon.

3.4 Sequence of Events

In the first part of the paper, we consider the following sequence of events:

At the very beginning of the time horizon the manufacturer decides on how many units of capacity she wants to reserve with her long-term supplier for the N periods and pays the associated reservation price, $R(K)$. Then, in every period, the manufacturer has first the option to purchase (up to $K$ units) from the long-term supplier knowing only her current inventory, the most recent spot price, and the distribution of the spot price in the current period (an assumption that we relax later in the paper). Once the spot price is announced, she can either purchase additional units on the spot market or reduce her inventory by selling on the spot market, including units that she just acquired from the long-term supplier. For example, after selling some or part of the items purchased via the options contract in the same period in the spot market, the manufacturer could immediately arrange for the items to be shipped to the new buyer by notifying her supplier about the new shipping address. Finally, demand realizes,
and holding and shortage costs have to be paid. The leftover inventory (which may be negative) becomes the starting inventory of the next period.

Figure 1 depicts the sequence of events from the manufacturer’s perspective, where we denote the optimal choice of long-term capacity by $K^*$ and refer to the long-term supply contract and the spot market as LT and SM, respectively.

**Remark 6** This sequence of events corresponds to reality in the utility industry, where call options on capacity are being used and the exercise decision is based in almost all cases on a conditional expectation of the spot price (e.g., the decision is made in the morning—based on the actual spot price at that time—for the afternoon, by which time the spot price may have changed dramatically).

**Remark 7** In Section 6, we generalize our results and modify the sequence of events such that the manufacturer has full information on the spot price realization in the period when making her exercise decision. Comparing the results of these two scenarios provides us with the value of spot price information.
3.5 Notation

Throughout this paper we use the following notation for the manufacturer’s decision problem:

- $i_{N+1}$: Starting inventory of the manufacturer
- $\pi_{N+1}$: Most recent spot price realization, i.e., initial information about the spot price
- $d_{N+1}$: Most recent demand realization at the beginning of the time horizon
- $i_{n+1}$: Inventory at the beginning of period $n$, i.e., at the end of period $n + 1$
- $\pi_n$: Spot price realization during period $n$
- $x_n$: Quantity bought from the long-term supplier in period $n$
- $y_n$: Quantity purchased/sold on the spot market in period $n$ with $y_n$ being negative in case the manufacturer sells part or all of her inventory on the spot market
- $E_{\pi_n|\pi_{n+1}}$: (Conditional) expectation of spot price in period $n$ given that the spot price realization in period $n + 1$ was $\pi_{n+1}$
- $E_{d_n|d_{n+1}}$: (Conditional) expectation of demand in period $n$ given that the demand realization in period $n + 1$ was $d_{n+1}$
- $h_n$: Unit holding cost in period $n$, $h_n \geq 0$
- $s_n$: Unit shortage cost in period $n$, $s_n \geq 0$
- $\alpha$: One-period discount factor, $\alpha \in (0, 1]$

**Remark 8** Note that although we do not explicitly account for this in our notation, we allow not only for correlation of the spot prices from one period to the next and for correlation of the demands in subsequent periods but also for correlation between the spot price and the next or previous realization of the demand, i.e., $E_{d_n|d_{n+1}} \triangleq E_{d_n|\pi_n,d_{n+1}}$ and $E_{\pi_n|\pi_{n+1}} \triangleq E_{\pi_n|d_{n+1},\pi_{n+1}}$.

**Remark 9** Note also that we denote by $\pi_n$ both the random variable and the realization
of the spot price in period \( n \).

4 Supply Options Contract

We consider a situation where the manufacturer has to first replenish her inventory via the long-term contract (knowing only the spot price distribution) and then (after the spot price realizes) the manufacturer can make additional purchases on the spot market. Finally, demand becomes known and holding and shortage costs have to be paid. The manufacturer’s decision problem in period \( n \) knowing that she owns options for \( K \) units (where \( n \in \{1, \ldots, N\} \) and \( n = 1 \) denotes the last period, i.e., \( n - 1 \) is the number of periods left in the time span) is thus as follows:

\[
G^K_{n+1}(i_{n+1}, \pi_{n+1}, d_{n+1}) = \min_{0 \leq x_n \leq K} \left\{ +E_{\pi_n|\pi_{n+1}} \left[ \min_{-(i_{n+1}+x_n) \leq y_n} \left\{ \begin{array}{l}
x_n p_n(K) \\
y_n \pi_n + s_n E_{d_n|d_{n+1}} \left[ \{d_n - i_{n+1} - x_n - y_n\}^+ \right] \\
+ h_n E_{d_n|d_{n+1}} \left[ \{i_{n+1} + y_n + x_n - d_n\}^+ \right] \\
+ \alpha E_{d_n|d_{n+1}} \left[ G^K_n(i_{n+1} + x_n + y_n - d_n, \pi_n, d_n) \right] \end{array} \right\} \right\}
\]

where \( s_n \) and \( h_n \) are the unit shortage and holding costs, respectively. \( E_{\pi_n|\pi_{n+1}} \) and \( E_{d_n|d_{n+1}} \) denote the (conditional) expectations of the spot price realization in period \( n \), \( \pi_n \), and the demand realization, \( d_n \), given that the spot price and the demand in the previous period were \( \pi_{n+1} \) and \( d_{n+1} \), respectively. In any period the manufacturer can buy at most \( K \) units from the long-term supplier and sell off at most her entire on-hand inventory or has to bring it at least back to zero if it was negative, hence \( y_n \geq -(i_{n+1} + x_n) \).

Remark 10 Notice that a negative value of \( y_n \) corresponds to the situation where the manufacturer selling part or all of her on-hand inventory in period \( n \).
Remark 11 Notice also that we could remove the constraint on $y_n$, the amount purchased on the spot market in period $n$, if we would allow the manufacturer to short sell, i.e., the manufacturer could borrow inventory from other players in the market and return it at the latest after the last period, $n = 1$.

At the end of the last period in the time span, i.e., $n = 1$, any leftover inventory is salvaged at the current spot price ($\pi_1$) and backlogged demand has to be satisfied via the spot market. Therefore,

$$ G^K_1(i_1, \pi_1, d_1) = -[i_1]^+ \pi_1 + [\pi_1]^+ \pi_1 = -i_1 \pi_1 $$

and the manufacturer’s overall objective function equals

$$ \min_{K \geq 0} [G^K_{N+1}(i_{N+1}, \pi_{N+1}, d_{N+1}) + R(K, N)] $$

where $R(K, N)$ is the cost of reserving $K$ units with the long-term supplier over the $N$-period time horizon.

4.1 Optimal Policy for the $N$-Period Horizon

Not surprisingly, given the form of the manufacturer’s objective function, we can show that an optimal purchase-up-to level exists. More specifically, we can determine the optimal purchasing/sales quantity on the spot market as well as the optimal decision regarding purchases from the long-term supplier.

Proposition 1 The optimal purchasing quantity on the spot market in period 1 equals

$$ y_1^* = F_{d_1|d_2}^{-1} \left( \frac{s_1 - (1 - \alpha) \pi_1}{s_1 + h_1} \right) 1_{s_1 \geq (1 - \alpha) \pi_1} - i_2 $$

and with $n > 1$,

$$ y_n^* = F_{d_n|d_{n+1}}^{-1} \left( \frac{\alpha E_{d_n|d_{n+1}}[E_{\pi_{n-1}|\pi_n}[\pi_{n-1}]] + s_n - \pi_n}{s_n + h_n} \right) 1_{\alpha E_{d_n|d_{n+1}}[E_{\pi_{n-1}|\pi_n}[\pi_{n-1}]] + s_n \geq \pi_n} $$

$$ - x_n^* - i_{n+1} $$
Hence, \( y^*_n \) is

- Linearly decreasing in the starting inventory in period \( n \), \( i_{n+1} \), and in the amount ordered from the long-term supplier in the same period, \( x_n \).

- A non-increasing function of the spot price realization in that period, \( \pi_n \), if for \( n > 1 \),
  \[
  \frac{\partial}{\partial \pi_n} E_{\pi_n|d_{n+1}} \left[ E_{\pi_{n-1}|\pi_n} [\pi_{n-1}] \right] \leq 1 \text{ or if } \pi_{n-1} \text{ is independent of } \pi_n.
  \]

- A non-decreasing function of the discount factor, \( \alpha \).

**Remark 12** Notice that the results of Proposition 1 and all similar results later on are meant “almost surely”, i.e., they hold with probability 1.

The above proposition is quite intuitive as it states that the optimal purchasing amount from the spot market in period \( n \), \( y^*_n \), decreases in the starting inventory in that period, \( i_{n+1} \), and the amount ordered from the long-term supplier in that period, \( x_n \), and both \( i_{n+1} \) and \( x_n \) may increase the manufacturer’s inventory just before the spot market purchase/sell decision. However, it is not intuitively obvious that \( y^*_n \) decreases linearly in \( i_{n+1} \) and \( x_n \). Likewise, common sense tells us that the manufacturer purchases less on the spot market when the spot price is high. Evidently, the result is valid if the spot prices in the two periods (\( n \) and \( n - 1 \)) are independent of each other. The third point in the above proposition is confirmed by common sense, as a high discount factor, \( \alpha \), makes future purchases more expensive in today’s money, which in turn induces the manufacturer to buy earlier rather than later in the time span. Consequently, the optimal inventory level before the demand is realized (consisting of the sum of the starting inventory, the optimal number of options exercised, and the optimal purchasing quantity in the spot market) is either equal to a “critical ratio”
term or equal to zero:

\[
i_{n+1}^* + x_n^* + y_n^* = F^{-1}\left( \frac{\alpha E_{d_n | d_{n+1}} [E_{\pi_{n-1} | \pi_n} [\pi_{n-1}]] + s_n - \pi_n}{s_n + h_n} \right) \mathbb{1}_{\alpha E_{d_n | d_{n+1}} [E_{\pi_{n-1} | \pi_n} [\pi_{n-1}]] + s_n \geq \pi_n}
\]

In both cases, the target inventory level in period \( n \) after purchasing via the options contract and buying or selling on the spot market is thus independent of how it has been reached, i.e., the “exercise” policy of the options contract is independent of the inventory problem. Moreover, we recognize a typical newsvendor solution in that the “critical ratio” term equals the underage cost per unit divided by the sum of the underage and overage cost per unit. If we denote the unit underage and overage cost in period \( n \) by \( c_n^u \) and \( c_n^o \), respectively, then

\[
c_n^u = s_n + \alpha E_{d_n | d_{n+1}} [E_{\pi_{n-1} | \pi_n} [\pi_{n-1}]] - \pi_n
\]

\[
c_n^o = h_n - \alpha E_{d_n | d_{n+1}} [E_{\pi_{n-1} | \pi_n} [\pi_{n-1}]] + \pi_n
\]

That means the effective overage and underage costs in period \( n \) account for the expected discounted gain or loss due to the change in spot prices from period \( n \) to period \( n-1 \).

**Remark 13** Notice that Proposition 1 relies on the assumption that allows the manufacturer to backlog demand for only one period, i.e., if the manufacturer does not carry enough inventory to meet all her demand in a period, she is required to buy at least enough from the long-term supplier and/or the spot market in the next period to bring her inventory level back to zero before the demand is announced in that period, regardless of the spot prices. An equivalent way of reaching the same result as in Proposition 1 would be to impose an additional condition on the spot price in that

\[
\Pr (\alpha E_{d_n | d_{n+1}} [E_{\pi_{n-1} | \pi_n} [\pi_{n-1}]] + s_n < \pi_n) = 0 \text{ for all } n > 1.
\]

This condition says that, regardless of today’s spot price realization, it is more expensive in expectation, to incur a shortage penalty and to purchase items in the spot market in the next period rather
than to buy the items in the market today. Thus, the additional condition would prevent the manufacturer from becoming a broker and from speculating in the spot market.

Due to the fact that the optimal purchasing/sales quantity on the spot market is linearly decreasing in the optimal number of options exercised in any given period, we are able to establish an easy form for the optimal exercise decision of the supply options.

**Proposition 2** \( \forall n, n \in \{1, \ldots, N\}, \)

\[
x^*_n = \begin{cases} 
K & \text{if } p_n(K) \leq E_{\pi_n|\pi_{n+1}} [\pi_n] \\
0 & \text{if } p_n(K) > E_{\pi_n|\pi_{n+1}} [\pi_n]
\end{cases}
\]

Hence, we can observe that the options contract is matched by the spot market in some sense. Indeed, the above proposition states that the manufacturer will exercise her options in period \( n \) only if the options contract is “in the money”, i.e., if the exercise price per unit is less than or equal to the expected spot price in that period. And, if this is the case then she will always choose to exercise all her options, regardless of the number of options she holds. Otherwise, the options contract is “out of the money” and she will not purchase anything from her supplier in period \( n \). Interestingly, the above result means that the supply options contract is a mere financial tool, independent of the manufacturer’s inventory level. Thus, the supply options contract is equivalent to a financial derivative with the spot price as underlying security and serves only as a hedge against spot price uncertainty. The main reason for this result to hold is that we assumed that the manufacturer can buy the full amount \( K \) (if the price is right) and then immediately sell any excess in the spot market for a presumed expected-value gain. However, we believe that this assumption is quite realistic, since it is easy for the manufacturer to immediately notify her supplier about any changes in the shipping address after she has sold part or all of the goods in question on the spot market and to have the supplier ship the items to the new buyer instead of to the manufacturer’s facilities.
4.2 Optimal Choice of Options

Let us denote the optimal level of supply options capacity purchased from the long-term supplier by \( K^* \). Notice that the inherent unit value of the options contract in period \( n \) equals \( \max \left( 0, E_{\pi_n | \pi_{n+1}} [\pi_n] - p_n (K) \right) \), i.e., is equivalent to a financial call option, since the manufacturer’s decision on whether or not to exercise depends only on whether the expected market price, \( E_{\pi_n | \pi_{n+1}} [\pi_n] \), exceeds the exercise price in that period, \( p_n (K) \). Thus, the value of the options contract hinges only on the spot price distributions and is independent of the manufacturer’s inventory problem. Therefore, we can determine the value of the options contract by just comparing its cost, i.e., the reservation price, \( R(K, N) \), to its expected benefit instead of computing the manufacturer’s total expected cost with the options contract and then comparing it to the expected cost the manufacturer would have to pay without the options contract.

The expected benefit to the manufacturer of having reserved \( K \) units with the long-term supplier in period \( n \) in terms of money at the beginning of the time horizon equals

\[
\alpha^{N-n} K E_{\pi_N | \pi_{N+1}} \left[ E_{d_N | d_{N+1}} \left[ E_{d_{N-1} | d_N} \left[ \ldots E_{\pi_{n+1} | \pi_n} \left[ \{ E_{\pi_n | \pi_{n+1}} [\pi_n] - p_n (K) \}^+ \right] \right] \right] \right]
\]

And therefore, the total expected benefit of the long-term contract to the manufacturer is \( K \sum_{n=1}^{N} \alpha^{N-n} E_{\pi_N | \pi_{N+1}} \left[ \ldots E_{\pi_{n+1} | \pi_n} \left[ \{ E_{\pi_n | \pi_{n+1}} [\pi_n] - p_n (K) \}^+ \right] \right] \). Define

\[
\tilde{A}(K, N) = \sum_{n=1}^{N} \alpha^{N-n} E_{\pi_N | \pi_{N+1}} \left[ \ldots E_{\pi_{n+1} | \pi_n} \left[ \{ E_{\pi_n | \pi_{n+1}} [\pi_n] - p_n (K) \}^+ \right] \right]
\]

(1)

Then, by comparing the contract’s expected benefit to the cost of entering it, i.e., \( R(K, N) \), we obtain that

**Theorem 1** \( K^* \) is given by

\[
K^* = \arg \sup \left( \tilde{A}(K, N) K - R(K, N) \right)
\]
and the maximum expected discounted value of the supply options contract to the manufacturer equals

\[ \tilde{A}(K^*, N) K^* - R(K^*, N) \]  

Clearly, the role of the options contract is to hedge against (spot) price uncertainty, essentially playing the role of a financial derivative on the spot price, independent of the inventory problem.

Based on Theorem 1 we can find the optimal reservation quantity for special cases such as when both \( p_n(K) \) and \( R(K, N) \) are non-increasing:

**Conclusion 1** \( \text{If } \forall n \in \{1, \ldots, N\}, \ p_n(K) \text{ and } R(K, N) \text{ are non-increasing in } K \text{ then } K^* = \infty. \)

For the general case, the sensitivity of \( K^* \) with regard to the discount factor, \( \alpha \), the number of periods, \( N \), and the relative spot market and long-term contract prices is similar to that in the case of stationary spot price distributions.

**Proposition 3** \( K^* \) is a non-decreasing function of the discount factor, \( \alpha \), the number of periods in the horizon, \( N \), and of the expected difference between the various spot and options exercise prices, \( E_{\pi_N|\pi_{N+1}} \left[ \ldots E_{d_{n+1}|d_{n+2}} \left[ \{ E_{\pi_n|\pi_{n+1}} [\pi_n] - p_n(K) \}^+ \right] \right] \).

### 4.3 Market/“Fair” Value of Supply Options Contract

Recall that it is impossible to speak of a market value for supply contracts in general as their value is directly linked to the buyer’s inventory problem, e.g., a contract specifying a large contract quantity has little value to a manufacturer facing a low demand level. Likewise, the variability of demand and the shortage or holding cost per unit may be
different for different companies and thus the same supply contract is valued differently by different companies.

Notice, however, that in our setting, the manufacturer’s purchasing decision regarding the long-term supply contract is independent of the inventory problem and thus is independent of her demand distribution if the spot price is independent of the demand (which is what we will assume in this section) or if the spot price distributions and anything that could influence them is common knowledge in the marketplace. Hence, any manufacturer would place the same value on the long-term supply contract (assuming they have the same information with regard to the spot price distributions) and thus we have found the “fair” or market value of the long-term supply contract, which can not be easily established in general, without the presence of a spot market. To simplify our notation, we define

$$\tilde{B}(K, N) = \sum_{n=1}^{N} \alpha^{N-n} E_{\pi_N|\pi_{N+1}} \left[ \ldots E_{\pi_{n-1}|\pi_n} \left\{ \pi_n \right\}^{+} \right]$$

(4)

**Theorem 2** If the spot price distribution is independent of the demand distribution then the market value of a long-term supply contract for $K$ units over the $N$-period horizon at exercise prices per unit of $p_n(K)$, $n \in \{1, \ldots, N\}$, equals $\tilde{B}(K, N) K - R(K, N)$.

**Remark 14** Note that $\tilde{B}(K, N)$ equals $\tilde{A}(K, N)$ when the spot prices and the demand realizations are independent.

The above theorem allows us to determine how the value of the options contract changes with changes in the discount factor, the length of the time span, and the exercise unit price.

**Proposition 4** If the spot price distribution is independent of the demand distribution then the value of the long-term supply contract for $K$ units is
• **Non-decreasing in the discount factor,** $\alpha$, **and the number of periods in the time horizon,** $N$

• **Non-increasing in the exercise prices per unit,** $p_n(K)$, **and the reservation price,** $R(K, N)$.

## 5 Supply Contract with Fixed Quantity Commitment

If the supply contract with the long-term supplier is not structured as a series of European call options but rather specifies a fixed amount that has to be purchased at every period, the manufacturer’s problem will be as follows:

The manufacturer has to first replenish her inventory via the long-term contract (without purchasing flexibility) and then (after the spot price is announced) the manufacturer can make additional purchases on the spot market. Finally, demand is realized and holding and shortage costs have to be paid. For a supply contract with $K$ units, the manufacturer’s decision problem in period $n$ (where $n \in \{1, \ldots, N\}$ and $n = 1$ denotes the last period) is thus as follows:

$$G^K_{n+1}(i_{n+1}, \pi_{n+1}, d_{n+1}) = Kp_n(K) + E_{\pi_n|\pi_{n+1}} \left\{ \min_{-(i_{n+1}+K) \leq y_n} \right. \begin{align*}
&y_n\pi_n + s_n E_{d_n|d_{n+1}} \left[ \{d_n - i_{n+1} - x_n - y_n\}^+ \right] \\
&+ h_n E_{d_n|d_{n+1}} \left[ \{i_{n+1} + y_n + x_n - d_n\}^+ \right] \\
&+ \alpha E_{d_n|d_{n+1}} \left[ G^K_n(i_{n+1} + x_n + y_n - d_n, \pi_n, d_n) \right] \\
\left. \right\} 
\right. \right. \right.$$ 

where $s_n$ and $h_n$ are again the unit shortage and holding costs, and $p_n(K)$ is the unit price specified in the long-term contract. $E_{\pi_n|\pi_{n+1}}$ as well as $E_{d_n|d_{n+1}}$ denote the conditional expectations of the spot price and demand realizations in period $n$, given that the spot price and the demand in the previous period was $\pi_{n+1}$ and $d_{n+1}$,
respectively. At the end of the last period, i.e., \( n = 1 \), any leftover inventory is salvaged at the current spot price \( (\pi_1) \) and backlogged demand has to be fulfilled via the spot market. Therefore,

\[
G^K_1 (i_1, \pi_1, d_1) = -[i_1]^+ \pi_1 + [-i_1]^+ \pi_1 = -i_1 \pi_1
\]

and the manufacturer’s overall objective function equals

\[
\min_{K \geq 0} \left[ G^K_{N+1} (i_{N+1}, \pi_{N+1}, d_{N+1}) + R_C (K, N) \right]
\]

where the long-term contract guarantees the delivery of \( K \) units per period over the \( N \)-period time horizon and \( R_C (K, N) \) is the associated reservation cost (with commitment). Again, we assume \( R_C (K, N) \) to be non-decreasing in \( K \) and that \( R_C (0, N) = 0 \).

### 5.1 Optimal Choice of Long-term Capacity

As the case of a supply contract with fixed quantity commitment is a simplified case of the supply options contract, we find results similar to those in the previous case. Again, we show that an optimal purchase-up-to level exists and we determine the optimal purchasing quantity from the spot market in the \( N \) periods. Let us denote the optimal level of reserved capacity with the long-term supplier in the case of a long-term contract with commitment by \( K^*_C \).

Again, we can determine the expected value to the manufacturer of purchasing \( K \) units from the long-term supplier in every period by comparing the cost of entering the contract to the expected benefit resulting from the contract. We first observe that the expected benefit of the long-term contract in period \( n \) in terms of money at the beginning of the time horizon equals

\[
K \alpha^{N-n} \left[ E_{\pi_N|\pi_{N+1}} \left[ E_{d_N|d_{N+1}} \left[ E_{\pi_{N-1}|\pi_N} \left[ \ldots E_{d_{n+1}|d_{n+2}} \left[ \left\{ E_{\pi_n|\pi_{n+1}} \left[ \pi_n \right] - p_n (K) \right\}^+ \right]\right]\ldots\right]\right]\right]
\]

\[
- K \alpha^{N-n} \left[ E_{\pi_N|\pi_{N+1}} \left[ E_{d_N|d_{N+1}} \left[ E_{\pi_{N-1}|\pi_N} \left[ \ldots E_{d_{n+1}|d_{n+2}} \left[ \left\{ p_n (K) - E_{\pi_{n+1}|\pi_n} \left[ \pi_n \right] \right\}^+ \right]\right]\ldots\right]\right]\right]
\]

\[
= K \alpha^{N-n} \left[ E_{d_N|d_{N+1}} \left[ E_{\pi_{N-1}|\pi_N} \left[ \ldots E_{d_{n+1}|d_{n+2}} \left[ E_{\pi_n|\pi_{n+1}} \left[ \pi_n - p_n (K) \right] \right]\right]\ldots\right]\right]\right]
\]
Hence, the total expected benefit to the manufacturer due to the long-term contract is
\[ K \sum_{n=1}^{N} \alpha^{N-n} E_{\pi | \pi_{n+1}} \left[ \ldots E_{d_{n+1} | d_{n+2}} \left[ E_{\pi_n | \pi_{n+1}} [\pi_n] - p_n (K) \right] \right]. \]
Define
\[ \tilde{A}_C (K, N) = \sum_{n=1}^{N} \alpha^{N-n} E_{\pi | \pi_{n+1}} \left[ \ldots E_{d_{n+1} | d_{n+2}} \left[ E_{\pi_n | \pi_{n+1}} [\pi_n] - p_n (K) \right] \right] \]
(5)

Then we can again compare the cost of entering into the supply contract with fixed quantity commitment per period, \( R_C (K, N) \), to the expected benefit the manufacturer achieves through it.

**Theorem 3** \( K^*_C \) is given by
\[ K^*_C = \arg \sup \left( \tilde{A}_C (K, N) K - R_C (K, N) \right) \]

Again, for a special case of the pricing parameters, \( p_n (K) \) and \( R_C (K, N) \), we can define the manufacturer’s best choice of the reservation quantity \( K^*_C \) more precisely:

**Conclusion 2** If \( \forall n \in \{1, \ldots, N\} \), \( p_n (K) \) and \( R_C (K, N) \) are non-increasing in \( K \) then \( K^*_C = \infty \).

Moreover, we determine the sensitivity of \( K^*_C \) with respect to the discount factor \( \alpha \), the number of periods \( N \), and the expected difference between spot market and contract exercise prices:

**Proposition 5** \( K^*_C \) is a non-decreasing function of the discount factor, \( \alpha \), of the expected spot prices, \( E_{\pi | \pi_{n+1}} \left[ \ldots E_{d_{n+1} | d_{n+2}} \left[ E_{\pi_n | \pi_{n+1}} [\pi_n] - p_n (K) \right] \right] \), and of the number of periods in the horizon, \( N \).

Again, if the spot prices are independent of the manufacturer’s demand then the supply contract with fixed quantity commitment would have the same value for other buyers as well and we could thus speak of a “market” value of the contract in a similar fashion as in Section 4.3 for the supply options contract.
Remark 15  Note that we could “value” any mix between the options contract and the supply contract with fixed commitment using this approach. For instance, a quantity flexibility contract as defined by Tsay (1999), with a minimum quantity commitment, $Q_{\text{min}}$, and a maximum order quantity, $Q_{\text{max}}$, could be split into two individual contracts. The first one would be a contract with quantity commitment over $Q_{\text{min}}$ units and the second an options contract over $(Q_{\text{max}} - Q_{\text{min}})$ units; the overall value of the quantity flexibility contract could be determined by adding up the value of the two individual contracts.

5.1.1 Value of Quantity Flexibility

We can calculate the expected value of the quantity flexibility by taking the difference between the expected value of the supply contract with options and of the contract with fixed quantity commitment.

Since the unit value of the contract with options any period $n$ is as least as high as the unit value of the supply contract with fixed quantity commitment, i.e.,

$$\{E_{\pi_n|\pi_{n+1}} [\pi_n] - p_n (K)\}^+ \geq E_{\pi_n|\pi_{n+1}} [\pi_n] - p_n (K)$$

there is a non-negative value of flexibility for the buyer.

Proposition 6  The expected discounted value per unit to the manufacturer of the contract’s flexibility regarding quantity equals

$$\sum_{n=1}^{N} \alpha^{N-n} E_{\pi_N|\pi_{N+1}} \left[ E_{d_{N+1}|\pi_{N+1}} \left[ E_{\pi_{N-1}|\pi_N} \left[ \ldots E_{d_{n+2}|\pi_{n+2}} \left[ \{p_n (K) - E_{\pi_n|s_{n+1}} [\pi_n]\}^+ \right] \ldots \right] \right] \right]$$

Notice, however, that if the spot price is stationary, independent of the spot price in other periods, and independent of the demand and if $p_n (K) = p (K)$ for all $n$ then the
contract’s quantity flexibility has no value since a necessary condition for the contract to be accepted by the manufacturer would be \( p(K) \leq E[\pi] \) implying that

\[
\sum_{n=1}^{N} \alpha^{N-n} (p(K) - E[\pi])^+ = 0
\]

Thus, in such a situation, the manufacturer would pay the same amount for both the options contract and the long-term supply contract with fixed quantity commitment as well as for any mix between the two.

However, if the manufacturer is slightly risk averse and includes a variance term with positive weight in her objective function (while maintaining the minimization objective), it appears likely that she would prefer to use the supply contract with commitment, as it would limit fluctuations of the unit price she pays (and thus reduce the variations of her procurement costs).

6 Increased Information Setting

In this section, we consider a situation where the manufacturer replenishes her inventory via the long-term contract after the spot price is announced (i.e., she has full information regarding the spot price), and the manufacturer can make additional purchases or sales on the spot market at the same time. At the end of the period demand is realized and holding and shortage costs have to be paid. After having reserved \( K \) options with her supplier, the manufacturer’s decision problem in period \( n \) (where \( n \in \{1, .., N\} \) and \( n = 1 \) denotes the last period) is thus as follows:

\[
G_{n+1}^K (i_{n+1}, \pi_{n+1}, d_{n+1})
\]

\[
= E_{\pi_n|\pi_{n+1}} \left[ \min_{0 \leq x_n \leq K} \left\{ x_n p_n (K) + \min_{-(i_{n+1}+x_n) \leq y_n} \left[ y_n \pi_n + s_n E_{d_n|d_{n+1}} \left\{ \{d_n - i_{n+1} - x_n - y_n\}^+ \right\} + h_n E_{d_n|d_{n+1}} \left\{ \{i_{n+1} + y_n + x_n - d_n\}^+ \right\} + \alpha \left[ E_{d_n|d_{n+1}} [G_n^K (i_{n+1} + x_n + y_n - d_n, \pi_n, d_n)] \right\} \right] \right\} \right]
\]
with $s_n$ and $h_n$ denoting the unit shortage and holding costs, respectively. $E_{\pi_n | \pi_{n+1}}$ and $E_{d_n | d_{n+1}}$ are the (conditional) expectations of the spot price realization in period $n$, $\pi_n$, and the demand realization, $d_n$, given that the spot price and the demand in the previous period were $\pi_{n+1}$ and $d_{n+1}$, respectively. Any leftover inventory at the end of the last period, i.e., $n = 1$, is salvaged at the current spot price ($\pi_1$) and backlogged demand has to be fulfilled via the spot market. Therefore,

$$G^K_1(i_1, \pi_1, d_1) = -i_1\pi_1$$

and the manufacturer’s overall objective function equals

$$\min_{K \geq 0} \left[ G^K_{N+1}(i_{N+1}, \pi_{N+1}, d_{N+1}) + R(K, N) \right]$$

with $R(K, N)$ referring to the cost of reserving $K$ units with the long-term supplier over the $N$-period time horizon.

See Figure 2 for a graphical illustration of this setting. Again, we denote the supply contract by LT and the spot market by SM. Since the newsvendor structure of the problem is preserved in this new setting, we can show that the optimal purchase-up-to levels and the optimal purchase quantities from the spot market in every period.
are the same as in the previous setting, i.e., all results of Proposition 1 apply directly. Furthermore,

**Proposition 7** The manufacturer’s best exercise decision regarding the options contract in period \( n \), \( x_n^* \), is to select

\[
x_n^* = \begin{cases} 
K & \text{if } p_n(K) \leq \pi_n \\
0 & \text{otherwise}
\end{cases}
\]

Hence, the additional information regarding the spot price realization does not change the structure of our results and intervenes only in the manufacturer’s exercise decision, which is now based on the actual spot price rather than the expected spot price as before. The inventory problem as expressed by the “critical ratio” term in the optimal purchasing quantity on the spot market is exactly the same as previously.

### 6.1 Optimal Choice of Options

Under the modified setting, the expected benefit resulting from a supply options contract for \( K \) units in period \( n \) in terms of money at the beginning of the time horizon equals

\[
\alpha^{N-n} KE_{\pi_N|\pi_{N+1}} \left[ E_{d_N|d_{N+1}} \left[ E_{\pi_{N-1}|\pi_N} \left[ \ldots E_{d_{n+1}|d_{n+2}} \left[ E_{\pi_n|\pi_{n+1}} \left[ \{\pi_n - p_n(K)\}^+ \right] \right] \ldots \right] \right] \right]
\]

Hence, the total expected benefit to the manufacturer due to the long-term contract equals \( K \sum_{n=1}^{N} \alpha^{N-n} E_{\pi_N|\pi_{N+1}} \left[ \ldots E_{d_{n+1}|d_{n+2}} \left[ E_{\pi_n|\pi_{n+1}} \left[ \{\pi_n - p_n(K)\}^+ \right] \right] \right] \). Define

\[
T(K,N) = \sum_{n=1}^{N} \alpha^{N-n} E_{\pi_N|\pi_{N+1}} \left[ \ldots E_{d_{n+1}|d_{n+2}} \left[ E_{\pi_n|\pi_{n+1}} \left[ \{\pi_n - p_n(K)\}^+ \right] \right] \right]
\]

Therefore,

**Theorem 4** The manufacturer will choose \( K^* \) is given by

\[
K^* = \arg \sup (T(K, N) K - R(K, N))
\]
6.2 Value of Spot Price Information

Using the results we obtained for the modified sequence of events, we can compute the expected value of the spot price information to the manufacturer as the difference between the expected discounted value in the two models. In general, we confirm common intuition in that there is a non-negative expected value to the manufacturer of having full information regarding the spot price.

Proposition 8 There is a non-negative expected value of information concerning the spot price, since \( \forall n \in \{1, \ldots, N\} \)

\[
E_{\pi_n|\pi_{n+1}} \left[ \{\pi_n - p_n(K)\}^+ \right] \geq \left\{ E_{\pi_n|\pi_{n+1}} [\pi_n] - p_n(K) \right\}^+
\]

More precisely, we can determine the expected discounted value per unit of having full information regarding the spot price equals

\[
\sum_{n=1}^{N} \alpha^{N-n} E_{\pi_N|\pi_{N+1}} \left[ E_{\pi_n|\pi_{n+1}} \left[ \{\pi_n - p_n(K)\}^+ \right] - \left\{ E_{\pi_n|\pi_{n+1}} [\pi_n] - p_n(K) \right\}^+ \right]
\]

Even for stationary spot price distributions, there is almost always a positive value of spot price information.

6.3 Supply Contract with Fixed Quantity Commitment

If the supply contract with the long-term supplier is not structured as a series of European call options but rather specifies at the very beginning of the time horizon a fixed amount that has to be purchased in every period, the manufacturer’s problem will be the same as in the previous setting since there is no exercise decision in each period and hence the information available to the manufacturer at the beginning of each period is immaterial. Hence, all results we obtained in the previous setting regarding the supply contract with fixed quantity commitment still hold and we proceed directly with discussing the value of flexibility in the modified setting.
6.3.1 Value of Flexibility

Again, we determine the expected value of supply flexibility by taking the difference between the expected discounted values of the supply contract with options and the contract with fixed quantity commitment.

Since in any period $n$, the unit value of the options contract is as least as high as the unit value of the supply contract with fixed quantity commitment, i.e.,

$$\{\pi_n - p_n (K)\}^+ \geq \pi_n - p_n (K)$$

there is again a non-negative value of flexibility for the manufacturer.

**Proposition 9** *The expected discounted value per unit to the manufacturer of the contract’s flexibility regarding quantity equals*

$$\sum_{n=1}^{N} \alpha^{N-n} E_{\pi_N|\pi_{N+1}} \left[ E_{d_{N+1}|d_{N+2}} \left[ E_{\pi_{N-1}|\pi_N} \left[ E_{d_{n+1}|d_{n+2}} \left[ E_{\pi_n|\pi_{n+1}} \left[ \{p_n (K) - \pi_n\}^+ \right]\right]\right]\right]\]$$

Even if the spot price is stationary is the expected value of contract’s flexibility regarding quantity in general not equal to zero.

6.4 Difference in Value of Flexibility between the Two Settings

With full information about the spot price, the expected unit value of quantity flexibility in period $n$ equals

$$E_{\pi_n|\pi_{n+1}} \left[ \{\pi_n - p_n (K)\}^+ - \{\pi_n - p_n (K)\} \right] = E_{\pi_n|\pi_{n+1}} \left[ \{p_n (K) - \pi_n\}^+ \right]$$

Without full information about the spot price, i.e., with only the (conditional) expectation of the spot price known at the time the decision is made regarding the number
of options to be exercised, the expected unit value of flexibility in period $n$ is

$$\{E_{\pi_n|\pi_{n+1}} [\pi_n] - p_n (K)\}^+ - \{E_{\pi_n|\pi_{n+1}} [\pi_n] - p_n (K)\} = \{p_n (K) - E_{\pi_n|\pi_{n+1}} [\pi_n]\}^+$$

Hence, by Jensen’s inequality, the expected value of flexibility is greater in the case of full information about the spot price, which reflects common intuition.

**Proposition 10** The expected value of flexibility is greater in the case of full information about the spot price and the difference in expected discounted value per unit between the two settings equals

$$\sum_{n=1}^{N} \alpha^{N-n} E_{\pi_N|\pi_{N+1}} [..E_{d_{n+1}|d_{n+2}} \{p_n (K) - \pi_n\}^+] - \{p_n (K) - E_{\pi_n|\pi_{n+1}} [\pi_n]\}^+]$$

7 Summary of Results & Outlook on Future Research

We presented a framework for analyzing the role and value of supply contracts (with or without options, i.e., quantity flexibility) in the presence of a spot market. We structure options contracts as a series of European call options over an $N$-period time horizon with a reservation price and pre-determined unit exercise prices that may vary from period to period.

Allowing for non-stationary demand and spot price distributions, which may be correlated between periods and between demand and spot price, we show that in the presence of a spot market, which is sufficiently liquid and which fulfills our no-arbitrage assumption, any supply contract becomes a mere financial tool, i.e., the role of the supply contracts is to hedge against (spot) price uncertainty, essentially becoming financial derivatives on the spot price, independent of the inventory problem. Notice that although we are considering a risk-neutral decision-maker, an additional benefit
of long-term contracts is that they may reduce the variability of the manufacturer’s procurement costs.

We derive the expected discounted value of the options contract to the manufacturer and compare it to the expected discounted value of a supply contract with fixed quantity commitment per period. Our analysis confirms common intuition in that there is no value of contract quantity flexibility if, in the original setting at the beginning of the paper, the spot price is stationary, independent between periods, and independent of the demand. In general, however, the manufacturer is willing to pay a premium for the quantity flexibility offered by the options contract.

Changing the state of information that the manufacturer has access to when deciding whether or not to exercise the options contract in a modified setting, we calculated the value of spot price information to the manufacturer. We also determine that the value of the contract’s flexibility regarding quantity is greater in the second setting.

The results reported here assume that both the spot market and the supply contracts have negligible delivery lead-times. A generalization with non-zero lead times for both the spot market and the long-term supplier is currently underway. Next, as mentioned in the introduction, we consider a Stackelberg game in a companion paper (Kleinknecht et al. 2002) where the supplier acts as the leader and sets the contract pricing parameters, $R$ and $p$, in anticipation of the manufacturer’s response. We could also study the impact of transactions costs, i.e., fixed ordering costs, for both channels (i.e., manufacturer and supplier). For instance, the manufacturer could have to pay a fixed amount every time she uses the spot market. A preliminary analysis suggests that introducing fixed transaction costs for purchases from the long-term supplier is quite simple, which is very good news since the finance literature has been struggling with transaction costs as the traditional replication argument fails in a situation with transaction costs. With transaction costs on the supply contract, $C_{LT}$, the effective unit price of the contract in period $n$ would equal $p_n(K) + C_{LT}/K$ and all our results
would directly apply. However, it seems that we may only be able to determine the structure of the optimal policy when fixed transaction costs occur for purchases or sales on the spot market and that we have to resort to numerical methods to study the value of supply options contracts in such a situation. As mentioned earlier in the paper, we could relax our assumptions regarding the demand and spot price distributions and allow for correlation between more than just subsequent periods or consider a different contract quantity, $K_n$, in every period $n$, and still use the same approach, i.e., without changing the nature of our results.

In addition, it would be interesting to study the impact of risk aversion on the decision-making by the manufacturer and the supplier. For instance, introducing a variance term in the respective objective functions would allow us to see whether options contracts can be used as a tool to hedge against uncertainty and to reduce variations in the procurement or sales process. This approach would be of great importance to procurement managers who are currently struggling with managing the risk reward in coping with the increase in demand and price uncertainty that is due to the creation of on-line marketplaces. Finally, we plan to consider other types of options, e.g., put options that would guarantee the supplier a minimum purchase quantity or American options that would provide an additional degree of flexibility in that the option could be exercised at any point in time within a given time window rather than at a specific instance. It appears that, contrary to common financial wisdom, exercising the (American) option at the end of the time horizon may not be optimal, i.e., “early exercise” may occur. However, we confirm common intuition in that purchasing all $K$ units reserved via the (American) options contract at the same point in time is an optimal policy.
References


