Managing Supply Chain Backorders under Vendor Managed Inventory: A Principal-Agent Approach and Empirical Analysis

Yuliang Yao
College of Business & Economics, Lehigh University
Email: yuy3@lehigh.edu, Tel: (610) 758-6726

Yan Dong
Carlson School of Business, University of Minnesota
Email: ydong@csom.umn.edu, Tel: (612) 625-2903

Martin Dresner
Robert H. Smith School of Business, University of Maryland
Email: mdresner@rhsmith.umd.edu, Tel: (301) 405-2204

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ABSTRACT

This paper examines the relationship between distributor backorder performance and its inventory levels under Vendor Managed Inventory (VMI) in a supply chain that consists of distributors and manufacturers. We construct a principal-agent model to show that distributors’ inventory managed by manufacturers can be used to induce distributor efforts, which are unobservable to the manufacturers, in converting lost sales into backorders in the case of stockouts. We use EDI transaction data collected from the electronic component and truck part industry to test our results from the analytical modeling. The empirical results provide strong supports for the proposition that there is a strong adverse relationship between the inventory level and backorder conversion rate, suggesting the manufacturer could use a lower distributor inventory as an incentive for the distributors to convert more lost sales into backorders.

Key Words: Vendor Managed Inventory; Backorders; Lost Sales; Incentive; Inventory
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1. Introduction

Interorganizational systems (IOSs) have become profoundly important in supply chain management as firms realize increasing needs for collaborations with their trading partners in order to respond to the fast-changing environments. IOSs provide an effective way to reduce the inefficiencies resulting from asymmetric information in supply chains (Bakos 1991). As a form of IOSs, with help of the rapid development of EDI and Internet technology, Vendor Managed Inventory (VMI) has gained tremendous attentions. Besides information sharing (see Kauffman and Mohtadi, 2003, for example), VMI also features a structural change of the relationships between upstream (say a manufacturer) and downstream (say a distributor) participants. The responsibility in deciding the timing and quantity for replenishing the distributor’s inventory has shifted from the distributor itself to the manufacturer. The shift of decision responsibility radically alters the contracting relationship, thereby providing a unique opportunity for the manufacturer to utilize VMI for greater benefits.

While it has been widely believed that VMI may help reduce supply chain inventory levels and stockouts, thereby generating cost savings, the messages from industry reports are mixed at best. Besides cost savings resulting from VMI, increasing market shares is another important attractiveness of VMI, especially when the distributor carries substitute products from competing manufacturers. One way to increase market shares is to convert lost sales into backorders in the case of stockouts.
Backorders are critical in measuring supply chain service performance (Hausman, 2002). As customer service has increasingly become a major goal of improving supply chain performance, how to manage backorders when stockouts occur has begun attracting more interests from both researchers and practitioners (for example, Lu, Song, and Yao, 2003; Song, 2002). With intensified competition in many industries—personal computers, electronics, automobiles, to name a few, the implication of backorders in maintaining and increasing market share in highly competitive markets has led companies to explore ways to control and manage backorders. In a supply chain where a distributor has multiple sources of supplies, it is to the manufacturer’s interests to prevent a customer from switching to its competitor’s products in the case of stockouts. This, however, may not be of the distributor’s interests, or may be indifferent to the distributor. The manufacturer need to find ways to assure that they do not lose customers and sales to their competitors once a stockout occurs. The widely adopted VMI offers a convenient and effective way to affect supply chain partners regarding better managing backorders.

The focus of this paper is to examine how the manufacturer may use VMI to extract (the best) effort from the distributor to convert lost sales into backorders in case of stockouts, i.e. increased market shares for the manufacturer. In the existing VMI literature, and supply chain literature in general, incentive contracts have been studied to investigate mechanisms regarding information sharing, inventory control, capacity allocation, etc. (e.g. Cachon, 2001, Cachon and Laveriere, 1999, Tsay, 2000, etc.). Backorders in supply chains have been analyzed mostly as an inventory control measure that is given with little strategic implications (e.g., Donohue, 2000). Lately, backorders have become the focal point of some supply chain research (Choi, Dai, and Song, 2003,
Lu, Song, and Yao, 2003, and Song, 2002), but these studies seek to minimize backorders to improve supply chain service. Our research takes a different approach and attempts to shed lights on how an upstream partner in a supply chain, given its stronger interests in backorder stockouts than lost sales stockouts, can provide incentives via VMI to influence its downstream partner’s behavior in order to increase (or maintain) its market shares. Our research intends to fill the gap through analytical modeling of the supply chain order replenishment process between VMI participants, and more importantly, through providing empirical evidence resulting from econometric analysis of the rich data gathered from the EDI transactions in supply chains of the electronic component and truck part industries.

While many have argued that the manufacturer under VMI will just “flood the floor” of the distributor’s warehouses that the manufacturer manages, and others, e.g., Fry et al. 2001, suggest an inventory ceiling being imposed in a VMI contact, we propose that, under VMI, there can exist an incentive offered by the manufacturer to lower the inventory levels at the distributor’s as a result of inducing effort from the distributor to convert lost sales to backorders. In this view, backorders become more of a strategic weapon for the manufacturer to enhance their market shares, whereas the distributor enjoys lower inventory levels resulting from making efforts to manage backorders. This proposition is supported by our results derived from a principal-agent model capturing backorders, stockouts, and inventory, under stochastic demand. It is also supported by the following empirical examination that the distributors with lower average inventory level tend to have higher percentages of backorder stockouts in total stockouts.
The rest of the paper is organized as follows. Section 2 presents a literature review on VMI; section 3 discusses the development of the analytical model; section 4 presents the empirical model; section 5 provides discussion of the findings from both analytical and empirical models; and section 6 presents conclusions, limitations, and future research.

2. Literature Review

There is a rich body of literature that studies VMI. These papers can be summarized into two streams, ones that take VMI as a given structure and study pre vs. post benefits (e.g., Raghunathan and Yeh 2001; Lee et al. 1999; Dong et al. 2001) and optimal operational policies (e.g. Cetinkaya and Lee 2000), the others that study the issues on the structural design of VMI (e.g., Fray et al. 2001).

Raghunathan and Yeh (2001) found that the implementation of continuous replenishment programs (CRP, a supply chain initiative akin to VMI) is beneficial to both manufacturers and retailers in terms of inventory reductions. Inventory reductions are affected by characteristics of consumer demand; that is, when demand variance increases, inventory reductions due to continuous replenishment programs decrease. In a similar approach, Aviv (2002) compared VMI with information sharing and collaborative forecasting, and auto replenishment (CFAR) under auto-regressive demand process, and showed that VMI and CFAR are more important as the demand process is more correlated across periods, and as companies are able to explain a larger portion of the demand uncertainty through early demand information. They findings reconfirm empirical evidence presented by Lee et al. (1999) in which they showed inventory turns
and stockouts are improved after implementation of CRP, using data collected from 31 grocery retail chains. In another empirical study of just-in-time practices, which are often used in conjunction with VMI, Dong et al. (2001) found that the benefits from JIT, in terms of inventory reductions, are most likely to flow to buyers rather than to suppliers.

Cetinkaya and Lee (2000) presented analytical model for coordinating inventory and transportation decision in VMI. They argued that the order release policy in use with VMI influences the level of inventory required at the vendor, thus directly affecting a supplier’s inventory costs. As a result, the actual inventory requirements at the vendor are partly dictated by the parameters of the shipment-release policy. Cheung and Lee (2002) also examined shipping strategies in VMI, and identified scenarios that inventory costs may be lower. Iyer and Bergen (1997) studies another similar program, Quick Response (QR), and found that a number of variables, such as service level, wholesale price and volume commitments, are instrumental to make QR profitable for both the manufacturer and retailer.

The second stream of literature examined VMI through a lens of contracting theory. These works focused on the design of the VMI. For example, Fry et al. (2001) studied VMI under a \((z, Z)\) type contract, and found the \((z, Z)\) type of VMI contract performs significantly better than traditional retailer managed inventory in many settings, but can perform worse in others. Choi et al. (2004) discussed a number of performance matrices that may be used to measure VMI’s performance. They showed that the backorders should also be taken into account while having considered inventory levels. Cachon (2001) examined a two-echelon supply chain with one supplier and \(N\) retailers, and found that the competitions among the retailers may lead to substantially higher costs.
than optimal. He further showed that a change of control, i.e. implementation of VMI contract, could lead to optimal supply chain performance. Plambeck and Zenios (2003) considered VMI in a principle-agent setting that the principal motivates the agent to control the production rate in a manner that will minimize the principal’s own total expect discounted cost by making payments contingent on the inventory level. Finally, Marayanan et al. (2002) examine a retailer and a supplier under a newsvendor setting. The retailer carries substitutable products from other suppliers, thereby resulting in different stockouts costs for the supplier and retailer. They derived conditions under which stocking decision should be transferred from retailer to supplier, i.e. an implementation of VMI.

There are a number of relevant papers that have examined incentive mechanism in supply chains in general, though not in VMI setting particularly. For example, Chen (2000) examines the design of compensation packages to the sales force in order to induce them to exert effort in a way that actually smoothes the demand process. Lariviere and Porteus (2001) demonstrate the price can be an effective contract term to achieve supply chain coordination in the newsvendor setting.

3. Analytical Model

3.1 Modeling Framework

We model the VMI in a principal-agent setting where the manufacturer acts as a principal and the distributor acts as an agent. Due to information sharing in VMI, the manufacturer has full knowledge about demand distribution as well as distributor inventory costs and policy. The manufacturer makes decision on the replenishment
quantity \((q)\) to the distributor under VMI, and implements a production policy of Make-to-Order. Without loss of generality, we assume a single item managed in the VMI between the manufacturer and the distributor, and the distributor also carries substitute products offered from competing manufacturers. The selling price for the item at the distributor is \(p\), purchase cost (i.e. the manufacturer’s selling price) is \(w\), and marginal production cost for the manufacturer is \(v\).

In VMI, although the manufacturer manages the distributor’s inventory, it is still the distributor who interacts directly with their customers. In the case of stockouts, the customers have three options, place a backorder, purchase a substitute item from the distributor supplied by a competing manufacturer, or to purchase products from a different distributor. The first scenario results in backorder stockouts, whereas the second and third scenarios result in lost sales stockouts to the manufacturer. If products from different manufacturers have similar margins, the distributor generally is indifferent between backorder stockouts and lost sales stockouts, as long as the lost sales are substituted by other purchases at the distributor. The manufacturer, on the other hand, prefers backorder stockouts to the lost sales stockouts (Cachon 2001; Narayanan et al. 2002). Since directing extra efforts in assuring customers is costly for the distributor, the manufacturer would have to provide incentives for the distributor when stockouts occur. When the margins are substantially different, the distributor needs even more incentives from the manufacturers who contribute smaller margins so as to not divert its customers to the other products.

The extent to which lost sales stockouts are converted into backorder stockouts depends on the distributor’s effort. For example, when stockouts occur, if the
distributor’s sales representatives could spend a certain level of efforts in assuring the customer on the manufacturer’s product quality, guaranteed fast replenishment, or free on-site delivery, it would increase the possibility that the customer decides to place backorders, rather than buy a substitute. We denote the distributor’s effort as \( e \), and the effort \( e \) is not observable to the manufacturer. There are two levels of effort, High (\( H \)) and Low (\( L \)). That is, \( e \in \left[ e^H ; e^L \right] \). We denote the percentage of backorder stockouts to total stockouts as \( \theta \), \( 0 \leq \theta \leq 1 \), which depends on the effort the distributor makes and is unknown \( \text{ex ante} \). We assume that \( \theta \) follows a conditional distribution \( f(\theta \mid e) \), and this distribution is common knowledge. We further assume that the distributor of \( f(\theta \mid e) \) satisfies first order stochastic dominance, suggesting “good” results are more likely to happen under high efforts. The distributor incurs an increasing cost for exerting the effort, \( c(e) \), and \( c'(e)>0 \) and \( c''(e)>0 \). Accordingly, \( c(e^H) > c(e^L) \).

The demand is stochastic with a distribution of \( g(x) \). Both backorder stockouts and lost sales stockouts incur penalty costs for both the manufacturer and the distributor (for the distributor, only when the sales are lost to a competing distributor). The unit penalty costs for backorder stockouts is normalized to equal its gross margin for both the manufacturer and the distributor for computation convenience without loss of generality. That is, for every unit in backorders, the manufacturer and distributor have to expedite them with zero profits. We assume the manufacturer has large enough capacity so that it can fulfill any amount of backorders immediately. The unit penalty costs for lost sales stockouts are \( l_a \) and \( s_a \) for the manufacturer and the distributor, respectively, and \( l_a > s_a \) since the manufacturer is penalized more severely than the distributor as the distributor may sell a substitute product. Finally, we assume the overstocked products are disposed
at costs of $H$ and $h$ for the manufacturer and distributor, respectively, and $H \leq h$ to ensure the distributor will not sell the overstocked products back to the manufacturer for profits to the manufacturer. These assumptions are consistent with those in Cachon (2002).

Therefore we can write the expected profit functions for the manufacturer and distributor:

\[
E_x \pi_M = (w - v)q - \int_0^q H(q - x)g(x)dx - \int_{\theta(x)}^\infty \theta(1 - \theta)(x - q)g(x)dx
\]  

(1)

\[
E_x \pi_D = \int_0^\infty \left[(px - wq - h(q - x))g(x)dx + \int_{\theta(x)}^\infty [(p - w)q - s_x (1 - \theta)(x - q)]g(x)dx - c(e)\right]
\]  

(2)

Under VMI contracting, the sequence of the game is as follows. First, the manufacturer offers a VMI menu contract, $(\theta, q(\theta))$, to the retailer, and the distributor decides to accept it or reject it. If the contract is rejected, the distributor’s profits are normalized to zero. If accepted, the distributor decides whether to make an effort. Then, a replenishment order is placed and the contract is executed. Finally, the demand is realized.

The manufacturer does not observe the distributor’s effort directly, and can only contract on the \textit{ex post} backorder ratio $\theta$, which depends on $e$, and realizes when an order is placed, and order quantity. Since the distributor is risk averse with regard to the replenishment quantity and its effort is not observable, moral hazard problems may exist, and the manufacturer need to provide incentives to motivate the distributor to exert higher level of efforts.

The incentive mechanism is stated as follows:

\[
\max_{q(\theta)} E_x \pi_M = \int_0^1 E_x \pi_M (q, \theta) f(\theta) |_{\theta^H} d\theta 
\]  

(3)

subject to
The objective function is the manufacturer’s expected profits. The individual rationality constraint (IR) reflects the minimum level of profits that the distributor would accept the contract. The incentive compatibility constraint (IC) states that the distributor will choose High effort that results in higher profits for the distributor.

3.2 Model Development and Analysis

We first analyze the conditions under which the optimal contract exists. We then develop the first best and second best results, which lead to the main result regarding the VMI contract. Define the partial derivatives with regard to $q$ for the manufacturer and distributor’s expected profit functions as:

$$\pi'_M = \frac{\partial \pi_M}{\partial q} = w + l_a (1 - \theta) - \left[H + l_a (1 - \theta)\right]G(q)$$

$$\pi'_D = \frac{\partial \pi_D}{\partial q} = (p - w)(1 - \theta) + s_b \theta + s_a (1 - \theta) - \left[p + h - (p - w)\theta + s_b \theta + s_a (1 - \theta)\right]G(q)$$

Since both profits functions are concave in $q$, these unconditional first order partial derivatives lead to both parties’ preferred order quantities. Define $q_M^*$ where $\pi'_M = 0$, and $q_D^*$ where $\pi'_D = 0$. The following result shows the condition under which a feasible contract is also optimal.

**Lemma 1.** For any feasible contract $q(\theta)$, it is optimal if and only if $\pi'_M \pi'_D \leq 0$.

This result indicates a necessary condition under which an implementable contract is optimal. Since the manufacture is the party to make the decision on $q$, he will choose
If $q_M^*$ also satisfies all the constraints, resulting in the unconstrained first best contract. However, if the IR constraint is violated by $q_M^*$, the manufacturer has to change the $q$ so that the distributor’s profits increase until the IR binds. If and only if $\pi_M^* \pi_D^* \leq 0$, the manufacturer may increase (when $q_M^* < q_D^*$) or decrease (when $q_M^* > q_D^*$) the $q$ to increase the distributor’s profits.

It can be shown that for $q_M^* > q_D^*$, 

$$G(q_D^*) = \frac{(p - w) + s_a (1 - \theta)}{p + h + s_a (1 - \theta)}$$

(6)

$$G(q_M^*) = \frac{(w - v) + l_a (1 - \theta)}{H + l_a (1 - \theta)}$$

(7)

Since $G(.)$ is a cumulative density function, $G(.)$ is increasing in $q$. Therefore, $q_M^* > q_D^*$ suggests $G(q_M^*) > G(q_D^*)$. Compare (6) with (7), and collect terms, we can deduce a sufficient and necessary condition that describes the relationships between $q_M^*$ and $q_D^*$. This leads to the lemma 2.

**Lemma 2.** When

$H(p - w) - (p + h)(w - v) > (w - v - H)s_a (1 - \theta) + (h + w)l_a (1 - \theta), q_M^* > q_D^*$; and when $H(p - w) - (p + h)(w - v) < (w - v - H)s_a (1 - \theta) + (h + w)l_a (1 - \theta), q_M^* < q_D^*$.

Considering $(w - v - H)s_a (1 - \theta) + (h + w)l_a (1 - \theta) > 0$, a sufficient (not necessary) condition for $q_M^* > q_D^*$ can be deduced: $\frac{H}{p + h} < \frac{w - v}{p - w}$. Since $H < h < p$, it can be further simplified to: $\frac{H}{p + h} < 0.5$. This result indicates that $q_M^* > q_D^*$ satisfies as long as the profit margin for the manufacturer ($w - v$) relative to the distributor’s profit margin ($p - w$) is large enough, i.e. larger than 50%.
While both situations may occur given combinations of different price and cost parameters, we focus on the situation where \( q_M^* > q_D^* \). We assume this relationship because of the presence of increasing degree of scale economies moving up supply chains in many industries. We observed in our data that the number of distributors (downstream supply chain), 89, is substantially greater than the number of manufacturers (upstream supply chain), 4, leading to this assumption. It appears more appropriate to focus on the \( q_M^* > q_D^* \) scenario although this reduces the applicability of the results. Furthermore, our empirical analysis uses data collected from electronic component and truck part industry. These numbers to certain extent validate \( q_M^* > q_D^* \), at least in the electronic component and truck part industry.

To follow the above discussions, we have \( \pi'_M \geq 0 \) and \( \pi'_D \leq 0 \), corresponding to \( q_M^* > q_D^* \). For the sake of convenience, we further restrict our analysis to \( \pi'_M > 0 \) and \( \pi'_D < 0 \). We argue that maintaining \( \pi'_M = 0 \) or \( \pi'_D = 0 \) for any \( \theta \) is neither realistic nor necessary, although we recognize the fact that this may affect the extent to which our results are generalized.

The following second order derivatives provide further characteristics of the optimal contract.

\[
\pi''_M = \frac{\partial^2 \pi_M}{\partial q^2} = -[H + l_q(1-\theta)]g(q) < 0
\]

\[
\pi''_D = \frac{\partial^2 \pi_D}{\partial q^2} = -(p + h - (p-w)\theta + s_q \theta + s_a(1-\theta)]g(q) < 0
\]
Therefore, we can determine that $\frac{\pi_M'}{\pi_D'}$ is increasing in $q$, and $\left|\frac{\pi_M'}{\pi_D'}\right|$ is decreasing in $q$, $\forall q \in (q^*_D, q^*_M)$, because 
\[
\frac{\partial}{\partial q} \left( \frac{\pi_M'}{\pi_D'} \right) = \frac{\pi_M'' \pi_D - \pi_M' \pi_D''}{(\pi_D')^2} > 0.
\]

We now develop the first best solution and the second best solution of the model. The first best solution is obtained by substituting the IR constraint into the objective function and ignoring the IC constraint, whereas the second best solution is obtained by substituting both the IR and IC constraint into the objective function. We introduce the following lemma on the first and second best solutions.

Define: $A_1 = w - \nu + l_a(1 - \theta)$; $A_2 = \left[H + l_a(1 - \theta)\right]$; $B_1 = (p - w) + s_a(1 - \theta)$; $B_2 = \left[p + h + s_a(1 - \theta)\right]$. 

**Proposition 1:** In the first best solution, $G(q^{FB}) = \frac{A_1 + \mu B_1}{A_2 + \mu B_2}$. In the second best solution, $G(q^{SB}) = \frac{A_1 + \mu B_1 + B_2 \lambda \left[1 - \frac{f(\theta|e^t)}{f(\theta|e^u)}\right]}{A_2 + \mu B_2 + B_2 \lambda \left[1 - \frac{f(\theta|e^t)}{f(\theta|e^u)}\right]}$.

From this result, the importance of order quantity, as an incentive instrument to extract high effort from the distributor, depends on the conditional distribution of the backorder conversion ratio given effort. The following results further explore the relationship between the conditional distributions $f(\theta|e^u)$ and $f(\theta|e^t)$ for any feasible solutions to the problem.
Lemma 3: $q^{SB} \in (q^*_D, q^{FB})$ if $1 - \frac{f(\theta|^{L})}{f(\theta|^{H})} > 0$; and $q^{SB} \in (q^{FB}, q^*_M)$ if $1 - \frac{f(\theta|^{L})}{f(\theta|^{H})} < 0$.

As having shown earlier that $\pi'_M > 0$ and $\pi'_D < 0$ in order to satisfy $\frac{\pi'_M}{\pi'_D} < 0$, all feasible $q$ in the equilibrium solutions, including the first best $q^{FB}$ and the second best $q^{SB}$, should be in the range that $q \in (q^*_D, q^*_M)$. Hence, we can deduce that the first best solution should be also in the range that $q^{FB} \in (q^*_D, q^*_M)$. Since at the first best solution, IR constraint binds, suggesting that $\pi'_D(q^{FB}) = 0$. Considering $\pi'_D < 0$ that higher $q$ leads to lower retailer’s profit (i.e., $\pi'_D$), we can conclude that $\pi'_D(\tilde{q}) < 0$, for any $\tilde{q} \in (q^{FB}, q^*_M) > q^{FB}$, which violates the IR constraint that the retailer only participates when its profit is equal to or greater than 0. Therefore, any feasible second best $q^{SB}$ needs to be in the range that $q^{SB} \in (q^*_D, q^{FB})$.

Based on the possible range in which the second best $q^{SB}$ may reside and lemma 3, we can determine that for any second best solutions, the inequality of $1 - \frac{f(\theta|^{L})}{f(\theta|^{H})} > 0$ should always be satisfied. Furthermore, we assume Monotone Likelihood Ratio Property (MLRP) holds. MLRP is a property that ensures higher production level is clear evidence that the agent has made higher effort (Laffont and Martimort, 2002, page165). In our case, it suggests a higher effort level increases the likelihood of a high conversion rate more than the likelihood of a low conversion rate.
**Proposition 2:** Assume MLRP holds, \( \theta \) decreases in \( q \), i.e. \( \frac{\partial \theta}{\partial q} < 0 \), \( \forall q \in (q^*_D, q^{FB}) \).

Proposition 2 shows that in the optimal contract, the backorder conversion rate is negatively associated with the replenishment quantity, which determines the distributor’s inventory level directly. The lower the inventory levels the higher the backorder stockouts conversion rates. This result suggests that the manufacturer may use lowering inventory level as an incentive to the distributor to spend higher efforts when interacting with their customers in the case of stockouts to convert greater lost sales stockouts into backorder stockouts.

### 4. Empirical Model

#### 4.1 Methodology

We develop an empirical model based on the proposition derived from the analytical modeling in the last section. Proposition 2 shows that there is a negative relationship between stockouts conversion rate (\( \theta \)) and the replenishment quantity \( q \) (i.e. distributor’s inventory level). Inventory levels have aggregate effects on total stockouts (i.e. backorder stockouts + lost sales stockouts) that the lower the inventory levels the higher the total stockouts. Proposition 2 suggests, however, that the manufacturer may be able to provide the distributor with some inventory reductions as an incentive to induce the distributor’s high efforts in creating higher conversion rates (i.e. greater backorder stockouts and fewer lost sales stockouts given a certain level of total stockouts). This effect is above and beyond the aggregate effects. For example, suppose the total stockouts are 10 units with 5 of each for backorder and lost sales stockouts (\( \theta = 50\% \)). If
the manufacturer decides to reduce the replenishment quantity (i.e. distributor’s average inventory), then the total stockouts may increase to 12 units but with a combination of 7 and 5 units for backorder and lost sale stockouts, respectively ($\theta = 58.3\%$). The overall two-unit increase reflects the aggregate effects, whereas the conversion rate increase from 50% to 58.3% reflects the effects resulting from the distributor’s high effort. Our empirical model is constructed to test the existence and magnitude of the latter effects.

In the empirical study, we test the high effort effects using an interaction term of total stockouts and inventory with properly controlling for total stockouts and other exogenous variables. Our goal is to show that the marginal changes of backorder stockouts with regard to total stockouts are higher when inventory levels are lower, other things being equal. The reason that we do not directly test the changes of $\theta$ on inventory is because the difficulties in calculating $\theta$ when the sum of backorder stockouts and lost sales stockouts is zero as a denominator. Our empirical results are aimed at providing partial evidence that the incentive mechanism is at work, and the manufacturer can improve its backorder stockouts with lower levels of inventory.

Thus, we formulate our main empirical research question as:

*Inventory levels (INV) have a negative impact on the number of backorder stockouts (BS) for a given level of total stockouts (TS).*

Empirically, we focus on the equation $BS = g(TS; TS \times INV, Exogenous Variables)$, and the details are discussed in the following section.

4.2 Data

Firm level data were collected from Enterprise Data Management (EDM), a 3rd party logistic solution provider specialized in VMI. EDM helps suppliers and distributors
manage their inventory by integrating them through VMI processes. Distributors share
their inventory status, demand, and sales information with their manufacturers through a
standard EDI protocol, EDI 852, and in return, the manufacturers decide the time and the
quantity to replenish distributors’ inventory.

Data of 4 manufacturers managing 89 distributors’ inventory was collected. Among 4
suppliers, 2 of them are in the electronic component industry with one
managing 65 distributors’ and the other managing 4; the rest of 2 are in the truck parts
industry with one managing 6 and the other managing 14. The source data contains
weekly information for the most recent 8 weeks at the time the data were collected, i.e.
from the week of May 12, 2002 to the week of June 30, 2002 and for the oldest 8 weeks
which is two years ago since only two year’s data was kept in their systems, i.e., from the
week of July 30, 2000 to the week of September 17, 2000, on inventory level, backorder
stockouts, lost sales stockouts, total items managed, and sales at distributor’s. Annual
sales for each distributor are also collected.

Data were aggregated over 8 weeks for both most recent and the oldest period,
respectively, so that it becomes a two period cross sectional data set. Among 89
distributors, 10 of them started VMI practice between the oldest period and the most
recent period. The 10 observations for the oldest period were not used in our analysis as
we focuses on the distributors under VMI practices. Hence, the data set contains 168
(89*2-10) observations.

Backorder stockouts are the number of days per week that backorders occurred,
and backorder stockouts occur when requested quantity is greater than on hand inventory
and the customer agrees to place a backorder. Lost sales stockouts are the number of days
per week that lost sales occurred, and lost sales stockouts occur when requested quantity is greater than on hand inventory and the customer either purchases partial requested quantity, or decides not to purchase at all. In order to better understand how the two types of stockouts occurred and recorded by the distributors, we present a hypothetical case in the following table.

**Table 1: Backorder Stockouts and Lost Sales Stockouts**

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Backorder Stockouts</th>
<th>Lost Sales Stockouts</th>
<th>Distributor’s Record</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. The customer wants the whole 120 units backordered.</td>
<td>Yes</td>
<td>No</td>
<td>Backorder Stockouts</td>
</tr>
<tr>
<td>2. The customer takes 100 units on hand, and wants the rest of 20 units backordered.</td>
<td>Yes</td>
<td>No</td>
<td>Backorder Stockouts</td>
</tr>
<tr>
<td>3. The customer takes the 100 units on hand and then buys the rest of 20 units from a competitor.</td>
<td>No</td>
<td>Yes</td>
<td>Lost Sales Stockouts</td>
</tr>
<tr>
<td>4. The customer buys all of 120 units from a competitor.</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
</tr>
</tbody>
</table>

It can be seen from the table that the distributor’s data correctly recorded scenario 1-3, but failed on scenario 4, meaning that reported lost sales stockouts would be lower than actual lost sales stockouts. We assume that unrecorded lost sales stockouts happens randomly across the distributors, so that the effect of under accounted data may wash off.

4.3 Estimation and Results

The regression model specified in section 4.1 is estimated in two stages due to the endogeneity between inventory and total stockouts. The first stage is an estimation of inventory as an instrumental variable. The second stage estimates the backorder stockouts equation with fitted value of inventory generated from the first stage instrumental
estimation. Since backorder stockouts are the counted number of days that backordering events occur during a week, it follows Poisson distribution. Ordinary Least Squared (OLS) estimation is not appropriate, and Poisson family regressions, such as Poisson regression or negative binomial regressions, are considered (Greene 1997).

Therefore, the estimation equations can be further formulated as follows:

\[
INV = \alpha_0 + \alpha_1 ITEMS + \alpha_2 SALES + \alpha_3 SIZE + \alpha_4 TIME + \sum_{i=1}^{3} \alpha_i'MF_i
\]

\[
BS = \beta_0 + \beta_1 TS + \beta_2 TS*INV + \beta_3 ITEMS + \beta_4 SALES + \beta_5 TIME + \sum_{i=1}^{3} \beta_i'MF_i
\]

Where:

- Total Stockouts \((TS)\) is the counted days of total stockouts for all items managed, including backorder stockouts and lost sales stockouts, during a week.
- Backorder Stockouts \((BS)\) is the counted days of backorder stockouts for all items managed during a week.
- Inventory level \((INV)\) is the average on-hand quantity for all items during a week, average over 8 weeks.
- Weekly Sales \((SALES)\) is the total quantity sold for all items during a week, average over 8 weeks.
- Total Items \((ITEMS)\) is the total number of items managed by a manufacturer at a distributor’s location, average over 8 weeks.
- Annual Sales \((SIZE)\) is the dollar amount of annual sales for a distributor.
- Time Dummy \((TIME)\) is the dummy variables with 1 indicating the most recent data, and 0 indicating the oldest data.
• Manufacturer Dummies (MF) is the dummy variables created to control the fixed effects of different manufacturers.

• $\alpha, \beta, \alpha',$ and $\beta'$ are parameters to be estimated.

Table 2 presents the descriptive statistics.

**Table 2: Descriptive Statistics (N=168)**

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>S.D.</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Stockouts</td>
<td>1,015.66</td>
<td>1,138.22</td>
<td>2</td>
<td>5,189</td>
</tr>
<tr>
<td>Backorder Stockouts</td>
<td>197.78</td>
<td>485.08</td>
<td>0</td>
<td>3,047</td>
</tr>
<tr>
<td>Inventory ($)</td>
<td>248,319.5</td>
<td>464,257.4</td>
<td>926.99</td>
<td>3,419,705</td>
</tr>
<tr>
<td>Weekly Sales ($)</td>
<td>32,465.76</td>
<td>66,762.40</td>
<td>51.96</td>
<td>368,024</td>
</tr>
<tr>
<td>Total Items</td>
<td>589.61</td>
<td>547.22</td>
<td>10</td>
<td>2,928.25</td>
</tr>
<tr>
<td>Annual Sales (x10^6 $)</td>
<td>1.68</td>
<td>3.52</td>
<td>0.003</td>
<td>21.40</td>
</tr>
</tbody>
</table>

The two equations are estimated in two stages using STATA. The first equation is estimated using OLS. The second equation is first estimated using Poisson regression. The following goodness-of-fit test of the regression is significant (Goodness-of-fit chi2 = 12184.27; Prob > chi2(159) = 0.0000), suggesting an overdispersion of the dependent variable so that Poisson regression is inappropriate. Poisson regression assumes that the expected mean should be equal to the expected variance, which is violated in our case. Therefore, a more flexible form of Poisson family regression, negative binomial regression, is used. Table 2 presents the estimation results.

In the first equation, the coefficients for total items ($\alpha = -448.26$) and for weekly sales ($\alpha = -3.03$) are positive and significant ($p<0.001$). The results show that the higher the number of total items in VMI and the higher the weekly sales, the greater the
inventory level at the distributors’. The adjusted R-squared of 0.81 indicates a terrific fit in predicting instrumental variable of inventory.

In the second equation of backorder stockouts, the interaction term, calculated by multiplying fitted value of inventory from the first equation with total stockouts, is plugged in to test high effort effects. The coefficient for the interaction term \( \beta = -9.40 \times 10^{-9} \) is negative and significant \((p<0.01)\), indicating that inventory has adverse effects on the marginal changes of backorder stockouts with regard to total stockouts. The coefficients for total stockouts \( \beta = 0.00054 \) is positive and marginally significant \((p<0.10)\), indicating that changes of total stockouts are positively associated with backorder stockouts.

Plugging the estimated coefficients into the first order derivative of the backorder stockouts regression with respect to the total stockouts, we have:

\[
\frac{\partial BS}{\partial TS} = 0.00054 + (-9.40 \times 10^{-19}) TS \left( \frac{\partial INV}{\partial TS} \right) - 9.40 \times 10^{-19} INV
\]

We can conclude that \( \frac{\partial BS}{\partial TS} \left( INV^{High} \right) < \frac{\partial BS}{\partial TS} \left( INV^{low} \right) \). The result shows that distributors with lower inventory have higher marginal changes of backorder stockouts with regard to total stockouts. In addition, Chi-squared statistic for the second equation is highly significant. The Log Likelihood Indexes, which are equivalent to R squared in OLS, are 0.12, indicating a reasonable fit for the negative binomial regression analyses.
Table 3: Two-Stage Regression Results  
(Standard Errors in Parentheses)

<table>
<thead>
<tr>
<th>Dependent Variables</th>
<th>Inventory</th>
<th>Backorder</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Intercept</td>
<td>-55000.85</td>
<td>-0.36</td>
</tr>
<tr>
<td></td>
<td>(42090.55)</td>
<td>(0.63)</td>
</tr>
<tr>
<td>Total Stockouts</td>
<td></td>
<td>0.00054</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.00032)</td>
</tr>
<tr>
<td>TS* fitted INV (x 10^-9)</td>
<td>-9.40**</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(3.25)</td>
</tr>
<tr>
<td>Total Items</td>
<td>448.26***</td>
<td>0.004***</td>
</tr>
<tr>
<td></td>
<td>(46.78)</td>
<td>(0.0008)</td>
</tr>
<tr>
<td>Weekly Sales</td>
<td>3.03***</td>
<td>0.000015+</td>
</tr>
<tr>
<td></td>
<td>(0.55)</td>
<td>(8.27e-06)</td>
</tr>
<tr>
<td>Annual Sales</td>
<td>0.004</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.010)</td>
<td></td>
</tr>
<tr>
<td>TIME</td>
<td>-83477.97**</td>
<td>-4.36***</td>
</tr>
<tr>
<td></td>
<td>(31188.54)</td>
<td>(0.35)</td>
</tr>
<tr>
<td>Manufacture 1</td>
<td>-38956.83</td>
<td>3.10***</td>
</tr>
<tr>
<td></td>
<td>(44877.63)</td>
<td>(0.58)</td>
</tr>
<tr>
<td>Manufacture 2</td>
<td>64349.19</td>
<td>3.99***</td>
</tr>
<tr>
<td></td>
<td>(91238.64)</td>
<td>(0.90)</td>
</tr>
<tr>
<td>Manufacture 3</td>
<td>34166.74</td>
<td>1.77*</td>
</tr>
<tr>
<td></td>
<td>(69250.56)</td>
<td>(0.82)</td>
</tr>
</tbody>
</table>

Model Statistics

<table>
<thead>
<tr>
<th></th>
<th>168</th>
<th>168</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td></td>
<td></td>
</tr>
<tr>
<td>F Statistics</td>
<td>105.22***</td>
<td></td>
</tr>
<tr>
<td>Log Likelihood</td>
<td></td>
<td>-613.41</td>
</tr>
<tr>
<td>Chi-Squared</td>
<td></td>
<td>163.91***</td>
</tr>
<tr>
<td>Adjusted R2</td>
<td>0.81</td>
<td></td>
</tr>
<tr>
<td>Pseudo R2 (LRI)</td>
<td></td>
<td>0.12</td>
</tr>
</tbody>
</table>

+ p < 0.10; * p < 0.05; ** p < 0.01; ***p<0.001

4. Scenario Analysis
We present a scenario analysis to better illustrate the adverse effect inventory has on backorder conversion rate based on the estimated coefficients in table 3. We assume it is the old time, manufacturer #1 (i.e., Time=0; Manufacturer 1 = 1; Manufacturer 2 = 0; Manufacturer 3 = 0), and total items and weekly sales are at their mean values. We identify 9 scenarios representing all possible combinations of 3 different values of total stockouts and inventory. 3 different values of total stockouts represent high (2000), medium (1016, the mean of total stockouts), and low (500), and 3 different values of inventory represent high (3419705, the max value of the inventory), medium (248320, the mean of inventory), and low (927, the min value of inventory). Table 4 presents the scenario analysis.

It can be seen from the table that backorder conversion rate ($\theta$) is negatively associated with inventory level for a given level of total stockouts. For example, in the case of total stockouts of 1016 units, the backorder conversion ratio reduces from 45.01% to 4.24% when inventory level increases from 927 to 248320, and to 0% when inventory level further increases to 3419705. This result provide further evidence in supporting our proposition that the manufacturers are able to use lower inventory to induce the distributors to exert higher effort in converting lost sales stockouts into backorder stockouts, as measured by stockouts conversion rates.

It is also interesting to note that backorder conversion rate is also decreasing in total stockouts for a given level of inventory. For example, backorder conversion rate reduces from 69.52% when total stockouts are 500 units, to 45.01% when total stockouts are 1016 units, and to 39.24% when total stockouts are 2000 units.
Table 4: Numerical Examples

<table>
<thead>
<tr>
<th>Total Stockouts</th>
<th>Inventory ($)</th>
<th>Backorder Stockouts (Units)</th>
<th>Backorder Conversion Rate (θ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>500</td>
<td>927</td>
<td>348</td>
<td>69.52%</td>
</tr>
<tr>
<td>500</td>
<td>248320</td>
<td>109</td>
<td>21.73%</td>
</tr>
<tr>
<td>500</td>
<td>3419705</td>
<td>0</td>
<td>0.00%</td>
</tr>
<tr>
<td>1016</td>
<td>927</td>
<td>457</td>
<td>45.01%</td>
</tr>
<tr>
<td>1016</td>
<td>248320</td>
<td>43</td>
<td>4.24%</td>
</tr>
<tr>
<td>1016</td>
<td>3419705</td>
<td>0</td>
<td>0.00%</td>
</tr>
<tr>
<td>2000</td>
<td>927</td>
<td>785</td>
<td>39.24%</td>
</tr>
<tr>
<td>2000</td>
<td>248320</td>
<td>7</td>
<td>0.37%</td>
</tr>
<tr>
<td>2000</td>
<td>3419705</td>
<td>0</td>
<td>0.00%</td>
</tr>
</tbody>
</table>

5. Concluding Remarks

In this paper we examine an incentive mechanism design in VMI that the manufacturer can use to increase its market shares. In particular, we show that the manufacturer can use VMI as a mechanism and use lower managed inventory as an incentive to induce the distributors to exert their best efforts in increasing the manufacturer’s backorders in the case of stockouts.

Our model follows the notions used in Cachon (2001) and Narayanan et al. (2002) that backorder stockouts and lost sales stockouts have different implications in costs for the manufacturer and the distributor. Lost sale stockouts hurts the manufacturer more than the distributor as the distributor usually carries substitute products from other competing suppliers. This cost implication gap motivates the manufacturer to design a contract of incentive with the distributor in order to mitigate the problem.

We first construct an analytical model in the principal-agent setting to show that the manufacture can offer a menu of combination of replenishment quantity (i.e., average inventory level) and backorder conversion rate that induces the distributor to choose high efforts in increasing the conversion rate. We then test the proposition we generated from
the analytical modeling through empirical modeling of firm data collected from the industry. Our empirical results provide strong supports for our proposition. We demonstrate there are a strong adverse relationship between the inventory level and backorder conversion rate, suggesting the distributors with lower inventory managed by the manufacturers work more “diligently” in converting lost sales into backorders, i.e. increased backorder conversion rate and market shares.

This paper contributes to the VMI literature, and supply chain literature in general, by approaching VMI differently in identifying a potential moral hazard problem that exists between participants. The research sheds lights on how an upstream partner in VMI, given its stronger preferences in backorder to lost sales, can provide incentive via VMI to impact its downstream partner’s behavior in order to mitigate the moral hazard problem, thereby increasing its market shares. More importantly, the research is one of very few papers that empirically tested derived proposition using industry data in the VMI literature. The results provide strong empirical evidence that VMI can be used as an incentive mechanism for the upstream participants to capitalize many benefits, such as increased market shares.
References


Appendix

Proof of Proposition 1

Substituting the IR constraint into the objective function and ignoring the IC constraint, we have:

\[ L(q, \mu) = \int_0^1 \pi_M f(\theta | e^H) d\theta + \mu \left[ \int_0^1 \pi_D f(\theta | e^H) d\theta - c(e^H) \right] \]  \tag{A-1}

Taking partial derivative with regard to \( q \) pointwise, we obtain the first order condition, and set it to zero, we have:

\[ \frac{\pi'_M}{\pi'_D} + \mu = 0 \]  \tag{A-2}

Since \( \frac{\pi'_M}{\pi'_D} < 0, \mu > 0 \), indicating IR is binding. Therefore, from (A-2) we can solve the \( q \) at equilibrium:

\[ G(q^{FB}) = \frac{A_1 + \mu B_1}{A_2 + \mu B_2}. \]

The second best solution is obtained by substituting both the IR and IC constraint into the objective function. Thus, we have:

\[ L(q, \lambda, \mu) = \int_0^1 \pi_M f(\theta | e^H) d\theta + \lambda \left[ \int_0^1 \pi_D \left[ f(\theta | e^H) - f(\theta | e^L) \right] d\theta - c(e^H) + c(e^L) \right] + \mu \left[ \int_0^1 \pi_D f(\theta | e^H) d\theta - c(e^H) \right] \]  \tag{A-3}

Taking partial derivative with regard to \( q \) pointwise, we obtain the first order condition, and set it to zero, we have:

\[ \frac{\pi'_M}{\pi'_D} \left[ 1 - \frac{f(\theta | e^L)}{f(\theta | e^H)} \right] + \mu = 0 \]  \tag{A-4}
Since \( \mu = -E(\frac{\pi'_M}{\pi'_D}) = \int_0^1 \frac{\pi'_M}{\pi'_D} f(\theta|e^H) d\theta > 0 \), indicating the IR binds.

Inserting \( \mu = -E(\frac{\pi'_M}{\pi'_D}) = \int_0^1 \frac{\pi'_M}{\pi'_D} f(\theta|e^H) d\theta \) into (A-4), and multiplying it by \( f(\theta|e^H)\pi_D \), we obtain:

\[
\lambda [f(\theta|e^H) - f(\theta|e^L)] \pi_D = f(\theta|e^H)\pi_D \left[ \frac{\pi'_M}{\pi'_D} - E(\frac{\pi'_M}{\pi'_D}) \right] (A-5)
\]

Integrating (A-5) over \( \theta \in [0,1] \), we have:

\[
\int_0^1 \lambda \pi_D [f(\theta|e^H) - f(\theta|e^L)] d\theta = 1 \pi_D \left[ \frac{\pi'_M}{\pi'_D} - E(\frac{\pi'_M}{\pi'_D}) \right] f(\theta|e^H) d\theta (A-6)
\]

From the slackness condition of (7), we know that:

\[
\lambda \int_0^1 \pi_D [f(\theta|e^H) - f(\theta|e^L)] d\theta = \lambda [c(e^H) - c(e^L)] (A-7)
\]

Substituting (A-7) into (A-6), we have:

\[
\lambda [c(e^H) - c(e^L)] = \int_0^1 \pi_D \left[ \frac{\pi'_M}{\pi'_D} - E(\frac{\pi'_M}{\pi'_D}) \right] f(\theta|e^H) d\theta = \text{Cov}(\pi_D, \frac{\pi'_M}{\pi'_D}) \geq 0 (A-8)
\]

Hence, \( \lambda \geq 0 \) since \( \frac{\pi'_M}{\pi'_D} \) and \( \pi_D \) vary in same direction over \( q \). Also, \( \lambda = 0 \) only if \( q^{SB}(\theta) \) is a constant, but in this case the IC is necessarily violated. As a result, we have \( \lambda > 0 \), indicating IC binds.

Therefore, from (14) we can solve the \( q \) at equilibrium:

\[
G(q^{SB}) = \frac{A_1 + \mu B_1 + B_1 \lambda \left[ 1 - \frac{f(\theta|e^L)}{f(\theta|e^H)} \right]}{A_2 + \mu B_2 + B_2 \lambda \left[ 1 - \frac{f(\theta|e^L)}{f(\theta|e^H)} \right]} (A-9)
\]
Q.E.D.

**Proof of Lemma 3**

Remember \( q^{FB} \) is the q that satisfies \( \mu = -\frac{\pi'_M (q^{FB})}{\pi'_D (q^{FB})} \). Inserting this into (A-4), we have:

\[
\frac{\pi'_M (q^{FB})}{\pi'_D (q^{FB})} - \frac{\pi'_M (q^{SB})}{\pi'_D (q^{SB})} = \lambda \left[ 1 - \frac{f(\theta e^I)}{f(\theta e^H)} \right]
\]

(A-10)

We know that \( \frac{\pi'_M}{\pi'_D} \) is increasing in q. If \( 1 - \frac{f(\theta e^I)}{f(\theta e^H)} > 0 \), then

\[
\frac{\pi'_M (q^{FB})}{\pi'_D (q^{FB})} - \frac{\pi'_M (q^{SB})}{\pi'_D (q^{SB})} = \lambda \left[ 1 - \frac{f(\theta e^I)}{f(\theta e^H)} \right] > 0. \text{ Hence, } q^{SB} \in (q^*_D, q^{FB}).
\]

It is easy to see the proof for the rest. Q.E.D.

**Proof of Proposition 2**

Set (A-4)=0, we have:

\[
\pi'_M + \lambda \pi'_D \left[ 1 - \frac{f(\theta e^I)}{f(\theta e^H)} \right] + \mu \pi'_D = 0
\]

Let’s write: \( F(\theta, q) = \pi'_M + \lambda \pi'_D \left[ 1 - \frac{f(\theta e^I)}{f(\theta e^H)} \right] + \mu \pi'_D = 0 \)

Using implicit function theorem:

\[
\frac{\partial \theta}{\partial q} = -\frac{\partial F/\partial q}{\partial F/\partial \theta} = -\frac{\mu \pi''_D + \lambda \pi''_D \left[ 1 - \frac{f(\theta e^I)}{f(\theta e^H)} \right] + \pi'_M}{\mu \frac{\partial \pi'_D}{\partial \theta} + \lambda \left[ \frac{\partial \pi'_D}{\partial \theta} \left[ 1 - \frac{f(\theta e^I)}{f(\theta e^H)} \right] + \pi'_D \frac{d}{d \theta} \left[ 1 - \frac{f(\theta e^I)}{f(\theta e^H)} \right] \right] + \frac{\partial \pi'_M}{\partial \theta}}
\]
Where:

\[
\frac{\partial \pi'_M}{\partial \theta} = -l_a + l_a F(q) < 0
\]

\[
\frac{\partial \pi'_D}{\partial \theta} = -(p - w) + s_b - s_a + [(p - w) + s_b - s_a] F(q) < 0
\]

\[
\pi''_M = \frac{\partial^2 \pi_M}{\partial q^2} = -[H + l_a (1 - \theta)] f(q) < 0
\]

\[
\pi''_R = \frac{\partial^2 \pi_R}{\partial q^2} = -[p + h - (p - w)\theta + s_b \theta + s_a (1 - \theta)] f(q) < 0
\]

From corollary 1, we know that \(\left[ 1 - \frac{f(\theta|e^L)}{f(\theta|e^H)} \right] > 0\). Since MLRP holds, we have

\[
\frac{d}{d\theta} \left[ 1 - \frac{f(\theta|e^L)}{f(\theta|e^H)} \right] \geq 0.
\]

Therefore it is easy to see that \(\frac{\partial \theta}{\partial q} < 0\). Q.E.D.