Interplay Between Organic Listing and Sponsored Bidding in Search Advertising

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Abstract

This paper aims to explore the effects of organic listing as a competing information source on the advertising competition (i.e., sponsored bidding) and the outcome performances in search advertising. We set up a game-theoretic model in which firms bid for sponsored advertising slots and compete for consumers in the product market. Firms are asymmetrically differentiated in terms of market preference and are placed at organic slots with different prominence based on their relative popularity. We suggest that when facing two competing lists, leading firms’ sponsored bidding incentive is mainly preventive, whereas small firms’ sponsored bidding incentive is mainly promotive. We show that these two incentives change in the opposite directions when the difference in advertisers’ competitive strength decreases. As a result, even small firms may outbid stronger competitors and win good sponsored positions under such a co-listing setting. We further analyze the effects of the organic listing on equilibrium outcomes by comparing it with a benchmark case in which there is only a sponsored list. We find that organic listing compensates the leading firms to help smaller firms win better sponsored positions, which balances the equilibrium information structure through sponsored list without impairing the objectivity of the organic list. While organic listing may lower the search engine’s short-term revenue, it increases equilibrium consumer surplus, social welfare and sales diversity, which are in the long-term interest of the search engine. Finally, we suggest some possible direction to improve the performance of organic listing for highly asymmetric markets.

Keywords: Organic Listing; Sponsored Bidding; Search Advertising;
1 Introduction

Search advertising, in which advertisers bid for sponsored advertising slots listed on a search engine results page (SERP) alongside a list of organic (non-sponsored) links, has proven itself to be a successful revolution of traditional online and offline advertising. Internet search-related advertising is predicted to generate annual revenue over $45 billion worldwide by 2011, becoming the leading advertising medium (VSS, 2007). The huge industrial success has attracted increasing academic interest, which includes recent theoretical studies on advertisers’ sponsored bidding strategy and the optimal auction mechanism to sell the sponsored slots, and some empirical work investigating factors that affect the profitability of sponsored advertising. Nevertheless, the organic list, despite its being the origin and the major information source of search advertising, has been generally neglected in the literature. This paper aims to systematically analyze the effects of organic listing as a competing information source on the advertising competition (i.e., sponsored bidding) and the outcome performances in search advertising.

Two features deserve special attention in studying the role of organic listing. One is the unique information structure associated with a SERP; the other is the characteristics of the commonly used organic ranking mechanism.

In response to each user query of a particular keyword, the search engine returns a SERP that contains hyperlinks to websites related to the keyword. In practice, major search engines (e.g., Google, Yahoo!, and Bing) organize the SERP in a similar way. Two lists of links are paralleled: A list of non-sponsored links, or organic links, is placed in the wide column on the left, and a narrow column on the right (and sometimes a highlighted area on the top as well) contains the list of sponsored links. Organic links are ordered based on search engines’ proprietary ranking algorithm, and the ordering typically reflects different links’ relative relevance to the keyword. The sponsored list is composed of advertising slots for sale. They are usually sold via auction, in which the bidder with the highest bid (or score) wins the first sponsored slot, the second highest wins the second, and so on.

One distinction of the co-listing structure is that it creates two lists competing with each other for consumer attention. Figure 1 shows the result from an experiment tracking eye movement of users viewing Google search result pages (Hotchkiss et al., 2005). The reddest region indicates the highest attention level (i.e., 100%). As we can see, the top organic links attract the most attention (e.g., the
Figure 1: Eye movement of users viewing Google pages (Hotchkiss et al., 2005)

top three organic links are viewed by almost all experiment participants), while the top sponsored link attracts a certain level of attention but could be less significant compared to its organic counterparts (e.g., the first sponsored link is viewed by about half of the participants). Notice that those merchant websites interested in sponsored advertising may also appear in the organic list and thus could get significant attention from the organic list without paying anything. In this sense, the organic list not only competes for consumer attention but also plays a dominating role in such competition. Then why would advertisers placed at prominent positions in the organic list still be willing to spend money on sponsored bidding? By creating a competing list to its revenue source (i.e., the sponsored list), is the search engine jeopardizing its own revenue?

In addition to the information structure of SERP, the organic ranking rule is another key element. Although kept private in most cases, organic ranking rules, such as Google’s PageRank-based ranking rule, are commonly believed to fairly reflect relative relevance or popularity of different websites by utilizing the inter-linking structure of websites and many other factors. Economic analysis also validates that websites’ relative quality or relevance is aligned with their equilibrium number of incoming links, which is consistent with the essence of the typical link analysis algorithm (e.g., PageRank algorithm) used by search engines (e.g., Katona and Sarvary, 2008). In other words, a website with greater popularity or relevance is generally given a better position with higher prominence in the organic list, and vice versa. Among those merchant websites that are potential sponsored advertisers, the typical organic ranking mechanism tends to favor the leading firms by giving them higher organic
ranking and allocating them a greater level of prominence. Under such asymmetric allocation of the organic prominence resource, how different advertisers react in the sponsored bidding and what the bidding outcome is become subtle questions. Will the leading firms take advantage of their pre-empted prominence advantage and patronize the sponsored advertising actively, or might small firms bid aggressively as a fight back for the disadvantage in organic listing?

Directly related to the organic ranking mechanism, another issue of interest is whether typical organic ranking promotes sales diversification. It has been well documented that in the ecommerce environment, millions of small firms and individual sellers are able to survive and flourish, and the overall sales diversity is greatly increased, all of which add to the richness and diversity of the online community and become the spirit of the dot-com era. As the leading online marketing medium and major information gatekeeper, search engines recognize their role in promoting small advertisers and increasing the sales diversity. For example, Eric Schmidt, CEO of the dominating search giant Google, describes the company’s mission as “serving the long tail.”\footnote{http://longtail.typepad.com/the_long_tail/2005/05/google_longtail.html} Nevertheless, given the fact that large-scale mainstream firms are normally given prominent positions in the organic list, is the typical organic ranking mechanism really promoting the “tails,” or just making the big bigger? If the latter, it would be appealing to think of any possible improvement of organic ranking to better achieve the goal of “serving the tails.”

Motivated by these intriguing issues, this paper intends to capture the unique feature of the co-listing structure (i.e., organic list attracts most attention) and the essential characteristics of the organic ranking mechanism (i.e., organic ranking aligns with relative popularity), so as to explicitly address the following research questions related to organic listing:

- What are advertisers’ bidding incentives for sponsored slots in the presence of the organic listing? How do such incentives differ across various advertisers, and what is the expected bidding outcome?

- How does organic listing affect the outcome performance, particularly social efficiency and search engine benefit? How does organic listing affect the resulting sales diversity, and how well does it serve the tails?

- In certain cases, is there any possible way to improve the organic ranking mechanism to benefit
both the search engine and the online society?

We consider a game-theoretic model in which firms in different organic slots compete for sponsored slots via auction and then compete for consumers in price after getting different sponsored slots. Firms are *asymmetrically differentiated* in terms of market preference. The mainstream firm is preferred by the majority of consumers in the market, while the niche firm is preferred by a small portion of the market. We model the organic ranking outcome in a way that the leading firm in terms of market preference gets a top organic position with high prominence advantage. Consistent with the experimental findings, we assume that the top several organic positions attract a fairly high level of attention while the first sponsored link attains less but still reasonable attention level. Prominence decreases rapidly from the top downward in both organic and sponsored lists. We investigate the equilibrium outcome of the bidding competition, in which the value of sponsored slots is *endogenously* determined in the pricing competition. We analyze the effects of organic listing on equilibrium outcomes (including social welfare, sales diversity, and search engine benefit) by comparing with a benchmark case with the sponsored list only (without the organic list). We then propose possible improvement of the organic ranking mechanism so that search engine benefit, welfare, and diversity can all be increased in certain cases.

In analyzing the bidding competition, we identify two interacting effects that drive advertisers to compete for sponsored listing in the presence of the organic list, namely, the *promotive* effect and the *preventive* effect. The promotive effect, which means a firm can promote its exposure by winning a prominent sponsored slot, decreases when the firm's organic prominence increases; in contrast, the preventive effect, that a firm can prevent its competitors from increasing their prominence by occupying the prominent sponsored position, increases as the firm gets a better position in the organic list. We find that firms’ equilibrium profit functions are submodular, that is, the marginal benefit of improving sponsored prominence changes in the same direction as firms’ relative competitive strength. As a result, we show that when the competing firms are relatively comparable to each other, the disadvantageous one bids aggressively and wins the prominent sponsored slot; when the market preference is highly asymmetric, the leading firm outbids its competitor, occupying prominent positions in both organic and sponsored lists.

Compared to the case with no organic list, organic listing subsidizes the leading advertisers in prominence for free to dilute their sponsored bidding incentive and to adjust the competence difference
among advertisers. In general cases with moderate levels of market asymmetry, the effects of such subsidy are two-fold. On the one hand, the co-listing structure induces weak advertisers to win better sponsored positions, which effectively increases their exposures without impairing the objectivity of the organic list. As a result, while keeping general search engine users satisfied, the co-listing structure improves the surplus of potential consumers (who are looking for product information), overall social welfare, and sales diversity. On the other hand, the free exposure from organic listing reduces advertisers’ sponsored bidding incentives, which reduces the search engine’s direct revenue. In this sense, organic listing serves as a balance between short-term and long-term benefit—sacrificing short-term revenue to enhance consumer surplus, total welfare and sales diversity, which could lead to better long-term growth.

While organic listing’s beneficial effects function well in general, they may malfunction when coming to highly asymmetric competition among advertisers. For small firms at the very tail facing strong competitors, their bidding incentive would be too low to win a prominent sponsored position. As a result, the search engine not only bears direct revenue loss but also fails to induce structural improvement of consumer surplus, social welfare and sales diversity. To explore possible ways of mitigating such shortcomings and better serving the tail, we generalize the typical popularity-based pure organic ranking and propose a mixed organic ranking mechanism, which probabilistically places less popular websites to a prominent position in the organic list rather than ranking strictly based on popularity. We show that introducing mixed organic ranking in a highly asymmetric market could improve the search engine’s short-term revenue, as well as consumer surplus, social welfare and sales diversity concurrently.

In addition to its substantive contribution to the literature on search advertising, this study also has notable theoretical contributions. Compared to traditional economics of advertising literature and recent studies related to search advertising, the substantially new understanding added by this paper, from theoretical perspective, lie in at least two aspects. First, we view organic list as an extra information source in addition to the advertising channel (i.e., the sponsored list), and we study how such a unique information structure affects advertisers’ advertising strategies as well as the equilibrium outcomes. Second, we consider advertising resources as differentiated and exclusive.

Organic listing serves as a competing information source added into the advertising campaign, which brings new perspectives into informative advertising studies. From the informative advertis-
ing perspective, advertising competition is essentially competition for information coverage (Bagwell, 2007). In typical informative advertising, advertising channels function as the major information sources that convey the information of firms’ existence, product details, and prices. In search advertising, however, a non-advertising channel (i.e., the organic list) coexists with the advertising channel, provides similar information, and even plays a dominating role as the major information source. Unlike sponsored positions, organic ranking and exposure levels are supposed to be out of advertisers’ control and thus exogenous. Moreover, the ordering of organic links and the resulting differences in organic exposure correlate with advertisers’ intrinsic competence (e.g., relative popularity in market preference). Such a unique source of information, which any traditional advertising channel can hardly resemble, naturally affects advertisers’ advertising strategies in a distinctive way, as is elaborated in this paper.

The advertising resource is exclusive in search advertising by nature, as different slots have very different prominence levels and only one advertiser can stay at the most prominent advertising position. Advertisers have to compete against each other for good advertising positions, and auction is naturally introduced to sell these positions. Consequently, a small difference in competence could mean the huge difference between winning or losing the best advertising resource. This feature compels us to model the very small difference in firms’ competence and to explicitly analyze the bidding outcomes. Traditional economic models of advertising, starting from Butters (1977), consider advertising technology in which advertisers independently decide advertising levels, which makes the equilibrium outcome less sensitive to the competence difference among firms. Naturally, symmetric competition remains the theme of traditional advertising literature (Grossman and Shapiro, 1984; Stegeman, 1991; Stahl, 1994). In contrast, we consider asymmetric competition among advertisers and incorporate asymmetric differentiation, as a combination of horizontal and vertical differentiation, into the model.

There is a large volume of literature on mechanism design and bidding strategies in sponsored auctions (Athey and Ellison, 2008; Edelman et al., 2007; Liu et al., 2010; Xu et al., 2010). As they consider the sponsored list only, we deepen the understanding of search advertising beyond these works by considering the interactions between the two lists. Another key distinction of our work is that most of these works treat the per-click value of a sponsored link as exogenously given (with Xu et al. (2010) among the few exceptions), while we endogenously investigate the valuation of sponsored
positions in price competition, connecting the product market competition with the sponsored bidding competition.

A limited number of studies focus on the role of organic listing in search advertising from different angles. Katona and Sarvary (2008) study the bidding patterns in search advertising when considering organic listing. Yang and Ghose (2010) are among the earliest to empirically investigate the potential synergistic effect between organic and sponsored links. This paper complements their works by systematically examining the effects of organic listing as an additional information source on advertisers’ bidding incentive for sponsored slots, search engines’ revenue, consumer surplus, social welfare, and sales diversities in equilibrium.

Some recent studies on referral intermediaries and recommender systems also provide relevant implications. Weber and Zheng (2007) develop an elegant model of search intermediary to study firms’ bidding strategies and the search engine’s optimal design, considering consumers’ search behavior. Their work relates to our study to the extent that with the optimal quality-weighting factor, the single list of sponsored positions considered in their model also exhibits certain features of organic listing in that the ranking partially conveys the information about advertisers’ relative performances. Nevertheless, since they look at a different question and focus on the sponsored list only, their model does not capture the unique information structure under the co-listing setting and thus does not consider the interaction between the difference in organic exposure and the response in sponsored bidding, which is the focus of our study. White (2009) studies the interaction between advertising and non-advertising lists from an interesting angle: by incorporating more firms in the non-advertising list, the intermediary can improve its quality to attract more users, but more firms in the lists bring down the market price (as a result of Cournot competition), lower the advertising firms’ profits, and may hurt the intermediary’s revenue. Since all positions in both lists are considered the same and one firm cannot appear in both lists, there is no informative interactions between the two lists and no bidding competition among advertisers in his model. Hagiu and Jullien (2010) carefully discuss why a recommender system would have incentive to recommend a “wrong” site to a consumer. Since firms do not compete for advertising positions, their model might be interpreted as studying how to deliver different manipulated organic lists to different users. Bhargava and Feng (2006) study an intriguing question of how to balance the numbers of organic and sponsored results in a single recommendation list. Not considering the information structure of the competing lists or the competition for advertising
positions, these two works differ from ours in the major focus.

The rest of the paper is organized as follows. In Section 2 we lay out the model. Section 3 derives the equilibrium pricing and bidding outcome. In Section 4, we first set up a benchmark case with no organic list and derive the corresponding equilibrium outcome. We then compare it with the equilibrium outcome derived under the regular case as in Section 3, to illustrate the effects of organic listing. As such analysis reveals a potential drawback of organic listing, we propose possible directions for improving organic ranking in Section 5. Section 6 concludes the paper with discussion on managerial implications.

2 The Model

We consider a search engine providing information about products. The search engine returns a SERP in response to each query of a particular keyword describing a certain type of product. Each SERP contains two lists of hyperlinks, namely, the organic list and the sponsored list. The organic list is composed of \( n \) organic links. These organic links are ranked by a proprietary algorithm designed by the search engine, in an order that reflects websites’ popularity and relevance to the keyword. The sponsored list contains \( s \) sponsored links. The slots for these links are sold via auction.

Consumers’ click behavior on each SERP is modeled in the following general way: When a firm’s link appears in only one of the two lists (e.g., in either the \( i \)th organic slot or the \( j \)th sponsored slot), an individual consumer clicks the \( i \)th organic link with probability \( \alpha_i \) and clicks the \( j \)th sponsored link with probability \( \beta_j \). When a firm’s link appears in both lists (e.g., in both the \( i \)th organic slot and the \( j \)th sponsored slot), an individual consumer clicks at least one of that firm’s links with probability \( 1 - (1 - \alpha_i)(1 - \beta_j)(1 - \gamma_{ij}) \). Without loss of generality, let \( \alpha_1 \geq \alpha_2 \geq ... \geq \alpha_n \) and \( \beta_1 \geq \beta_2 \geq ... \geq \beta_s \). We assume \( \beta_i < \alpha_i \), as it is generally believed that, compared to organic listing, consumers have a negative bias against the sponsored list due to the aversion to advertising. We use \( \gamma_{ij} \) to capture the potential synergistic effect between organic and sponsored listing in terms of attracting click-throughs. As is implied by the probability expression, \( \gamma_{ij} > 0 \) indicates complementary effect, whereas \( \gamma_{ij} < 0 \) implies substitute effect and \( \gamma_{ij} = 0 \) means no significant synergistic effect.

We consider two competing firms selling a certain type of product in the market.\(^2\) Their products

\(^2\)We focus on duopolistic analysis in the main body of the paper. In fact, as we can show, the qualitative results and main implications can be extended to oligopolistic competition.
are differentiated with asymmetric market preference. The firm whose products are favored by the majority of consumers or the mainstream of the market is denoted as \( M \); the firm selling products preferred by the minority of consumers or the niche market is termed as \( N \). Assume the two firms have the same production cost, which is normalized to zero.

There is a continuum of consumers with unit mass in the market. Each consumer has a unit demand of the product. Consumers differ in their taste. The majority of the market, with a portion \( 1 - \theta \) (\( 0 < \theta < \frac{1}{2} \)), prefer firm \( M \)'s product to \( N \)'s, while the others, with a proportion \( \theta \), prefer firm \( N \)'s product. We sometimes call the former \( M \)-type consumers and the latter \( N \)-type consumers. Consumers derive utility \( v \) from consuming their preferred product, and derive a discounted utility \( \tilde{k}v \) from the less preferred product, where \( \tilde{k} \) is uniformly distributed over \([0, 1]\) across all consumers. Without loss of generality, we normalize \( v \) to 1. Consumers are not aware of their preferences before the search process and therefore use the search engine to explore product information. On clicking the link of a firm, consumers visit the firm’s website, see the product information and the price, and learn their valuation of that firm’s product. We define a consumer’s net utility as the utility from consuming the product minus the price of the product. Consumers will purchase a product only when it generates a net utility exceeding a certain reservation value, which is normalized to zero. For those consumers who visit both firms’ websites, they purchase from the one giving a higher (positive) net utility.

In the organic list, firms are ranked in an exogenous order reflecting their market popularity or relevance to the keyword. In this sense, representing the mainstream of the product market, firm \( M \) is listed at the \( i_M \)th slot, which is around the very beginning of the organic list with a fairly high prominence level \( \alpha_{i_M} \). In contrast, firm \( N \) has a lower organic rank \( i_N \) with a lower prominence level \( \alpha_{i_N} \) (\( \alpha_{i_N} < \alpha_{i_M} \)). Note that such difference in position rank and prominence level could be significant due to the existence of other non-merchant links (e.g., Wikipedia entries and news sites) appearing in between. Since the idea here is that \( \alpha_{i_M} \) is close to 1 and \( \alpha_{i_N} \) is significantly less than \( \alpha_{i_M} \), for simplification of expression, we let \( \alpha_{i_M} = 1 \) and \( 0 < \alpha_{i_N} < 1 \). As we will see, the underlying spirit of this simplification is to highlight the established prominence advantage of the top organic link and the diminishing benefit of sponsored listing for the firm already occupying a prominent organic slot.

In the sponsored list, firms can bid for a prominent position. For all purposes of asymmetric duopoly analysis, let \( s = 2 \). The sponsored slots are sold via a second price auction, in which the firm
with the highest bid wins the first sponsored slot and pays an amount equal to the second highest bid. The firm with the lower bid stays in the second sponsored slot, paying a reserve price which is normalized to zero for simplicity. Here, we rank advertisers based on their bids on the total payment, which is in fact consistent with the common practice. In auctions of sponsored links, advertisers typically bid per-click unit prices and are ranked based on their per-click bids and the expected click-throughs on their sponsored links. Liu et al. (2010) shows that it is socially efficient to rank advertisers by the product of their per-click unit-price bids and their expected click-throughs, which exactly equals advertisers’ total willingness-to-pay. In our framework, both the search engine and the advertisers rationally anticipate the expected click-throughs on the links placed at different positions. As a result, advertisers make the per-click bidding decision in the same way as if they submit a total bid. For example, if a firm in the \( i \)th organic slot and the \( j \)th sponsored slot will attract \( \sigma_{ij} \) sponsored clicks, to win over another bidder with \( \sigma_{i'j'} \) expected sponsored clicks and per-click bid \( b' \), the firm has to bid pay-per-click \( b \) such that \( b \sigma_{ij} \geq b' \sigma_{i'j'} \) (i.e., its total willingness-to-pay has to exceed its competitor’s). Considering total bid rather than unit-price bid in our paper is simply to avoid unnecessary assumptions on the click-through of each sponsored link, and all the analysis and results remain unaffected.

In sum, the timing of the game is as follows: In the first stage, firms submit bids for the sponsored slot. In the second stage, after observing the bidding outcome, both firms set their price simultaneously. Finally, consumers browse the SERP, sample firms’ websites and make purchase decisions. Notice that consumers sample firm \( M \)'s website with probability 1, regardless of the sponsored bidding outcome. Consumers sample firm \( N \)'s website with probability \( 1 - (1 - \alpha_{i_N}) (1 - \beta_1) (1 - \gamma_{i_N1}) \) if firm \( N \) wins the first sponsored slot; otherwise, they sample its website with probability \( 1 - (1 - \alpha_{i_N}) (1 - \beta_2) (1 - \gamma_{i_N2}) \). To further simplify the notation, we define \( \psi_1 \equiv (1 - \alpha_{i_N}) (1 - \beta_1) (1 - \gamma_{i_N1}) \) and \( \psi_2 \equiv (1 - \alpha_{i_N}) (1 - \beta_2) (1 - \gamma_{i_N2}) \) and let \( 0 < \psi_1 < \psi_2 < 1 \), which means that winning the first sponsored slot increases the prominence level for firm \( N \).

\(^{a}(1 - \alpha_{i_N}) (1 - \beta_j) (1 - \gamma_{i_Nj}) > 0 \) because \( \beta_j < 1 \) (due to the discounting factor of advertising) and \( \gamma_{i_Nj} < 1 \) (due to \( \gamma_{i_Nj} = 1 \) representing the extremely positive synergy). \( (1 - \alpha_{i_N}) (1 - \beta_j) (1 - \gamma_{i_Nj}) < 1 \) is to exclude the unrealistic cases in which \( \gamma_{i_Nj} \) takes significantly negative values.
3 Equilibrium Analysis

In this section, we derive the equilibrium bidding outcome under the setting of co-listing structure that includes both organic list and sponsored links. In analyzing the equilibrium, we investigate the interplay in competing for sponsored links and provide rationale for different firms to decide their bidding strategies.

Along the line of backward induction, we start with the second stage price competition. We first formulate firms’ market shares under a complete information setting, where all of the consumers are aware of the two products and know the product details and price information:

\[
S_M (p_M, p_N) = (1 - \theta) \left(1 - [p_M - p_N]^+\right) + \theta [p_N - p_M]^+
\]

\[
S_N (p_N, p_M) = \theta \left(1 - [p_N - p_M]^+\right) + (1 - \theta) [p_M - p_N]^+
\]

where firms’ prices \(p_M, p_N \in [0, 1]\), and \([\cdot]^+\) represents \(\max\{\cdot, 0\}\). An \(M\)-type consumer will buy product \(M\) only if \(1 - p_M \geq \tilde{k} - p_N\), which explains the first term of \(S_M (p_M, p_N)\). The other terms can be interpreted in a similar way. Notice that consumers have their own preference of one product over the other, but the degree of their preference varies, and marginal consumers exist who are almost indifferent between the two products given the price difference. Such setting allows either firm, even the niche firm, to compete for market share against its competitor, as long as it can get enough exposure.

Similarly, we can define firms’ market shares in the case of informational monopoly where consumers are only aware of the one firm’s product information and its price.

\[
A_M (p_M) = (1 - \theta) + \theta \left(1 - p_M\right)
\]

\[
A_N (p_N) = \theta \left(1 - p_N\right) + (1 - \theta) \left(1 - p_N\right)
\]

for \(p_M, p_N \in [0, 1]\). In the case of \(A_M\), for example, all \(M\)-type consumers buy from firm \(M\), while \(N\)-type consumers buy from firm \(M\) only if \(\tilde{k} - p_M \geq 0\).

Recall that because of the co-listing structure of the SERP and consumers’ corresponding click behavior, consumers visit firm \(M\)’s website with probability 1. Consumers visit firm \(N\)’s website with probability \(1 - \psi_1\), if \(N\) wins the first sponsored link; otherwise, consumers visit \(N\)’s website with probability \(1 - \psi_2\), where \(0 < \psi_1 < \psi_2 < 1\). We simply denote \(1 - \psi\) as the probability of a
consumer’s visiting firm $N$’s website and

$$\psi = \begin{cases} 
\psi_1 & \text{when } N \text{ wins the first sponsored slot} \\
\psi_2 & \text{otherwise}
\end{cases}$$

Notice that $\psi$ can be interpreted as a measure of information incompleteness within the market, or it can be viewed as the level of informational dominance of the mainstream firm. A larger $\psi$ means that the mainstream firm has greater informational advantage over the niche firm, in the sense that a larger portion of consumers is unaware of the niche firm’s product. Winning the top sponsored slot can help the niche firm increase its exposure and improve the prominence level, by reducing $\psi$ from $\psi_2$ to $\psi_1$.

Based on the notations introduced above, we can now formulate firms’ demand functions for the given informational structure determined by the first stage bidding outcome (which is characterized by $\psi$). A proportion $\psi$ of consumers is aware of product $M$ only, and the other proportion is aware of both products. Therefore,

$$D_M (p_M, p_N) = \psi A_M (p_M) + (1 - \psi) S_M (p_M, p_N)$$
$$D_N (p_N, p_M) = (1 - \psi) S_N (p_N, p_M)$$

Firms’ profits can thus be written as

$$\pi_i (p_i, p_{\bar{i}}) = p_i D_i (p_i, p_{\bar{i}}), \{i, \bar{i}\} = \{M, N\}$$

Based on the best responses derived from maximizing the profit function, we can derive the equilibrium prices in the second stage.

$$\begin{cases} 
p_M^* = \min \{ \frac{2 - \theta (1 - \psi)}{3(1 - \theta) (1 - \psi) + 4 \theta \psi}, 1 \} \\
p_N^* = \frac{\theta + (1 - \theta) p_M^*}{2(1 - \theta)}
\end{cases}$$

Notice that in equilibrium, $p_N^* < p_M^*$; that is, the niche firm, which is at a disadvantage in terms of market preference, tends to cut its price to compete for market share against its stronger competitor. On the other hand, the mainstream firm tends to stay away from the intense price competition when
Table 1: Equilibrium Profit Functions in the Second Stage Price Competition

<table>
<thead>
<tr>
<th></th>
<th>$0 &lt; \psi &lt; \frac{1}{3}$</th>
<th>$\frac{1}{3} \leq \psi &lt; 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi^*_M (\psi, \theta)$</td>
<td>$\frac{[2-\theta (1-\psi)^2(1-\theta)-(1-2\theta)\psi]}{[3(1-\theta)(1-\psi)+4\theta \psi]^2}$</td>
<td>$(\frac{1}{2} - \theta) \psi + \frac{1}{2}$</td>
</tr>
<tr>
<td>$\pi^*_N (\psi, \theta)$</td>
<td>$\frac{[(1+\theta)(1-\theta)+\theta(3\theta-1)\psi]^2(1-\psi)}{(1-\theta)[3(1-\theta)(1-\psi)+4\theta \psi]^2}$</td>
<td>$\frac{1-\psi}{4(1-\theta)}$</td>
</tr>
</tbody>
</table>

it has significant informational advantage. In fact, $p^*_M = 1$ when $\psi \geq \frac{1}{3}$, according to Eq. (5). As long as its informational advantage is reasonably large, firm $M$ forgoes the price competition with $N$ and simply charges the monopoly price to fully exploit its guaranteed demand.

Next, we derive firms’ equilibrium profits from the second stage price competition, $\pi^*_i (\psi, \theta)$, as a function of the information incompleteness level $\psi$ and the market asymmetry level $\theta$. By substituting $\{p^*_M, p^*_N\}$ into Eq. (4), we summarize $\pi^*_i (\psi, \theta)$ in Table 1. Lemma 1 describes an important property of the equilibrium profit functions.

**Lemma 1.** Both firms’ equilibrium profit functions are submodular, that is, $\frac{\partial^2}{\partial \psi \partial \theta} \pi^*_i (\psi, \theta) < 0$, $i \in \{M, N\}$.

Lemma 1 reveals the trend that the marginal benefit of winning a better sponsored slot ($\frac{\partial}{\partial \psi} \pi^*_i (\psi, \theta)$) changes with the market structure ($\theta$). It is a crucial step toward the revelation of the bidding outcome, as we will discuss in detail soon. As we can show, $\frac{\partial}{\partial \psi} \pi^*_M (\psi, \theta) > 0$, indicating that firm $M$ benefits from enlarging its informational dominance by keeping its competitor’s prominence level low. Thus, the submodularity of $\pi^*_M (\psi, \theta)$ implies that such marginal benefit decreases as $\theta$ increases, or, in other words, as the two firms become more comparable. Likewise, generically, $\frac{\partial}{\partial \psi} \pi^*_N (\psi, \theta) < 0$, meaning that firm $N$ has incentive to reduce $\psi$ by winning a prominent sponsored slot, and the submodularity of $\pi^*_N (\psi, \theta)$ implies that such incentive increases as $\theta$ increases.

Based on the equilibrium profits in the second stage price competition, we next investigate the bidding competition in the first stage. In second-price auctions, it is well documented that bidding true value is a weakly dominant strategy for all bidders. Therefore, in the first stage, the unique
perfect equilibrium is that both firms bid their true value $b^*_i$:

$$\begin{cases}
  b^*_M = [\pi^*_M (\psi_2, \theta) - \pi^*_M (\psi_1, \theta)]^+ \\
  b^*_N = [\pi^*_N (\psi_1, \theta) - \pi^*_N (\psi_2, \theta)]^+
\end{cases} \quad (6)$$

which equals their respective equilibrium profit difference between winning the first sponsored slot and otherwise, bounded below at zero.

Applying the results from Lemma 1, we uncover the bidding outcome as follows.

**Proposition 1.** In equilibrium, there exists a cutoff $\theta^* (\psi_1, \psi_2)$, such that when $0 < \theta < \theta^* (\psi_1, \psi_2)$, $M$ bids higher and wins the first sponsored slot, and when $\theta^* (\psi_1, \psi_2) < \theta < \frac{1}{2}$, $N$ outbids its rival. Here, $\theta^* (\psi_1, \psi_2) \in (0, \frac{1}{2})$ and is defined by

$$\pi^*_M (\psi_2, \theta^*) - \pi^*_M (\psi_1, \theta^*) = \pi^*_N (\psi_1, \theta^*) - \pi^*_N (\psi_2, \theta^*) \quad (7)$$

Now we are able to provide reasonable answers to some of our initial research questions: What is a firm’s incentive to bid for a prominent sponsored slot when it is already placed at a prominent position in the organic list? How does organic listing affect firms’ bidding for sponsored links? How does the effect differ across differentiated firms?

Under the structure that organic list is paralleled with sponsored list, a prominent sponsored link benefits its winner in at least two aspects. The first aspect is the **promotive effect.** Winning a prominent sponsored slot increases a firm’s probability of being noticed via the additional sponsored click-throughs, and it may also create significant synergy between the organic and sponsored lists to further enhance the exposure for the firm. The second aspect can be viewed as the **preventive effect.** The firm that wins the prominent sponsored slot can keep its competitor away from that position and effectively prevent the competitor from increasing its exposure, and can thus reap its informational advantage.

As prominent organic links usually capture the most attention within a SERP, there is little room to improve exposure for those firms with top organic ranks. In this sense, those firms’ incentive of bidding for sponsored slots mainly originates from the preventive side. In contrast, a firm with a less prominent organic rank finds motive more from the promotive rather than the preventive
perspective, as winning a prominent sponsored link greatly complements its inadequate attention level from the organic list but can barely affect the high click-throughs attained by its competitors via their prominent organic positions. Therefore, as a firm’s organic rank improves, the promotive effect of the sponsored listing for the firm decreases and the preventive effect increases. We capture this trend in the inherent model setup, and further highlight the trend by letting $\alpha_{iM} = 1$ such that the top sponsored link has no promotive effect for firm $M$ and no preventive effect for firm $N$ accordingly. As mentioned before, this setup allows us to disentangle the two otherwise intertwined effects, tease out the interplay between the two effects in the bidding competition, and deliver neat insight on how such interaction evolves with the market structure.

In this framework, we can think of $\frac{\partial}{\partial \psi} \pi^*_M (\psi, \theta)$ as a measure of the marginal preventive effect of the sponsored listing, and consider $-\frac{\partial}{\partial \psi} \pi^*_N (\psi, \theta)$ measuring the marginal promotive effect. Lemma 1 shows the dynamic evolving of the two interacting effects when the market structure changes. As the market asymmetry decreases (i.e., $\theta$ increases), the marginal promotive effect of the sponsored listing increases and the marginal preventive effect decreases. This is because when the market preference becomes more diverse, the niche firm is more comparable to its competitor, and it can thus considerably improve its profit by increasing its prominence level via winning the prominent sponsored slot. In contrast, the market share that the mainstream firm can capture becomes less, even under informational monopoly, so the mainstream firm benefits less from blocking its competitor. Since the two effects evolve in the opposite directions as the market structure changes, naturally, there exists a threshold $\theta^*$, a certain cutoff level of market asymmetry, such that the promotive effect dominates when $\theta$ is above that threshold and the preventive effect dominates when $\theta$ is below that threshold, as is summarized by Proposition 1. Eq. (7) states that at the cutoff level of market asymmetry, the promotive benefit for $N$ equals the preventive benefit for $M$.

Proposition 1 also provides rationale for firms in different types of markets to determine the value of a prominent sponsored slot in bidding competition. In a highly asymmetric market, the leading firm should bid aggressively to win the top sponsored slot because occupying a prominent position in both the organic and sponsored list enlarges its informational dominance, ensures its advantageous position in price competition, and greatly improves its sales profit. However, when the market preference is relatively diversified, the firm at a disadvantage in terms of market preference should bid to win a prominent sponsored slot, especially when it is placed at a lower organic position with
an unsatisfactory attention level. This is because its marginal benefit from improving its prominence level is fairly high in this case.

Notice that the niche firm’s incentive to win the top sponsored link \((-\frac{\partial}{\partial \psi} \pi^*_N(\psi, \theta))\) decreases as \(\theta\) goes down. It is thus worth pointing out that in a certain parameter region, such incentive can fall so low that firm \(N\) is not willing to bid any positive amount.

**Corollary 1.** The niche firm may have no incentive for sponsored bidding; in particular, when \(\psi_2 < \frac{1}{3}\) and \(\theta < \frac{17 - \sqrt{145}}{24}\), \(b^*_N = 0\).

When the market preference is highly asymmetric (i.e., \(\theta\) is small), if the niche firm can gain a reasonable level of attention from the organic link and the second sponsored link (i.e., \(\psi_2 < \frac{1}{3}\)), it will not bother to bid for the top sponsored link at all. The reason is that should \(N\) win the first sponsored slot and increase its prominence level to \(1 - \psi_1\), it would trigger an intense price war with firm \(M\), which eventually results in an even lower equilibrium profit level for firm \(N\). Weak in competence, the niche firm is better off staying in a relatively low prominence level and therefore has no intention to bid for the top sponsored link at all. The driving force here is the unshakable prominence advantage given to the leading firm in the organic list.

### 4 Effects of Organic Listing

Having derived the equilibrium under the co-listing structure, in this section, we further investigate the effects of organic listing on the equilibrium outcomes. We first construct a case absent of organic listing as a benchmark, and then compare the equilibrium outcomes under the benchmark case with that derived from Section 3 in three aspects: overall social welfare, resulting sales diversity, and search engine benefit.

#### 4.1 A Benchmark Case

As a benchmark, we consider a case where each SERP contains the sponsored list only. The benchmark case can be imagined as the search engine’s choosing to display only one list of links, all of which are potential advertising slots to be sold. Similar practices can be found in some regional search engines, such as Baidu.com, the leading search engine in China, and in early versions of search advertising, such as those used by Goto.com and, later, Overture.com, in which there was only one
list, mixed with paid advertising links and organic search results, and advertisers could bid for their ranks. We call the original case with both organic list and sponsored list the co-listing case.

To be consistent with the original model, we consider two sponsored links on a SERP. An individual consumer clicks the first sponsored link with probability \( q_1 \) and the second with probability \( q_2 \) \((q_1 > q_2)\). To make the benchmark case comparable to the co-listing case, we let \( q_1 = 1 \) to model the dominant prominence of the first link on a webpage because it can capture the most user attention, just as the top organic link does in the co-listing case. Similarly to the co-listing case, we denote \( q_2 = 1 - \psi \), where \( \psi \) measures the level of information incompleteness or informational dominance.

All other settings (i.e., firms, market preference, auction rules) follow the original model in Section 2.

Similarly, we start the analysis from the second stage price competition. We can formulate firms’ demand functions when firm \( i \) wins the first sponsored slot while the other firm \( \bar{i} \) stays in the second, \( \{i, \bar{i}\} = \{M, N\} \):

\[
D_i(p_i, p_{\bar{i}}) = \psi A_i(p_i) + (1 - \psi) S_i(p_i, p_{\bar{i}}) \\
D_{\bar{i}}(p_{\bar{i}}, p_i) = (1 - \psi) S_{\bar{i}}(p_{\bar{i}}, p_i)
\]

(8)

where \( p_i \) and \( p_{\bar{i}} \) are the firms’ prices, and \( A_i(\cdot) \) and \( S_i(\cdot, \cdot) \) are defined by Eq.(2) and Eq.(1), respectively. When firm \( M \) wins the first sponsored slot and thus attracts most of the attention, as in the co-listing case, the demand function is exactly the same as before (see Eq.(3)). Therefore, both firms face the same competitive situation, and the equilibrium prices and profits remain in the same format as Eq.(5). The main difference, however, arises when firm \( N \) wins the first sponsored slot and firm \( M \) can only have the less-prominent sponsored position. Notice that the mainstream firm now could become informationally disadvantaged compared to its competitor, because it no longer has a guaranteed prominence dominance from occupying the top organic link as in the co-listing case. In other words, the niche firm can now win over significant prominence advantage to better compete for market shares. We derive the equilibrium prices when \( N \) wins the first sponsored position as follows.

\[
\begin{align*}
\hat{p}_M &= \frac{2 + \psi - \theta - \theta \psi}{(3 + \psi)(1 - \theta)} \\
\hat{p}_N &= \frac{1 + \psi + \theta - \theta \psi}{(3 + \psi)(1 - \theta)}
\end{align*}
\]

(9)

The equilibrium profit from the second stage price competition can be derived in a similar way, as
Table 2: Equilibrium Profits in the Second Stage Price Competition (Benchmark Case)

<table>
<thead>
<tr>
<th></th>
<th>When $M$ wins</th>
<th>When $N$ wins</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>M’s profit</strong></td>
<td>$\hat{\pi}_M = \frac{(1-\theta)(1-\psi)^2(1-\theta)-(1-2\theta)\psi}{3(1-\theta)(1-\psi)+4\theta\psi}$</td>
<td>$\hat{\pi}_N = \frac{(1-\theta)(1-\psi)^2(1-\theta)-(1-2\theta)\psi}{3(1-\theta)(1-\psi)+4\theta\psi}$</td>
</tr>
<tr>
<td><strong>N’s profit</strong></td>
<td>$\hat{\pi}_N = \frac{(1+\theta)(1-\theta)+(3\theta-1)\psi^2(1-\psi)}{(1-\theta)[3(1-\theta)(1-\psi)+4\theta\psi]}$</td>
<td>$\hat{\pi}_N = \frac{(1+\theta)(1-\theta)+(3\theta-1)\psi^2(1-\psi)}{(1-\theta)[3(1-\theta)(1-\psi)+4\theta\psi]}$</td>
</tr>
</tbody>
</table>

summarized by Table 2, where $\hat{\pi}_i^j$ is firm $i$’s equilibrium profit in the $j$th sponsored position in the benchmark case ($i \in \{M, N\}$, $j \in \{1, 2\}$).

In the first stage bidding competition, both firms bid their true value: $\hat{b}_i = [\hat{\pi}_i^1 - \hat{\pi}_i^2]_+$ ($i \in \{M, N\}$), which again is the difference between the equilibrium profits when they win the first sponsored slot and when they do not. By comparing two firms’ equilibrium bids $\hat{b}_M$ and $\hat{b}_N$, we can uncover the bidding outcome in the benchmark case.

**Proposition 2.** In the benchmark case, in equilibrium, $M$ always bids higher and wins the first sponsored slot; that is, $\hat{b}_M > \hat{b}_N$.

Proposition 2 shows that the only possible outcome of the bidding competition is that the firm with advantage in market preference wins the most prominent sponsored slot. Surprising as it might seem, this result can be well understood within the framework of the two aforementioned interacting effects. Recall that in the co-listing case, a prominent sponsored link engenders mainly preventative effect for the mainstream firm and mainly promotive effect for the niche firm. In contrast, in the benchmark case, winning a prominent sponsored slot engenders both promotive and preventative effects for either of the two firms. These two effects are amplified when a firm has greater competence in terms of market preference. As a result, as long as firm $M$ has market preference advantage over $N$ (i.e., $\theta < \frac{1}{2}$), firm $M$ always has greater incentive to win the top sponsored slot compared to firm $N$, regardless of the preference advantage magnitude.

It is also worth noting that in equilibrium both firms adopt the same pricing strategies as in the co-listing case (according to Eq. (5)) and firm $M$ charges a higher price than firm $N$.

**Corollary 2.** In the benchmark case, the niche firm has a positive bidding incentive in most cases; that is, when $\frac{5\sqrt{6}}{6} - 2 < \theta < \frac{1}{2}$, $\hat{b}_N > 0$. 

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Compared to Corollary 1, in the benchmark case, the niche firm bids a positive amount in a larger region of parameter value. This result also implies that firms’ bidding incentive could be greater in the benchmark case than in the co-listing case.

Next, we compare the equilibrium outcome in the co-listing case with that in the benchmark case and investigate the effects of organic listing on social welfare, sales diversity, and the search engine’s benefit. To focus on systematic difference rather than perplexing discussions on parameter values, we let $\psi = \psi_2$ so that we focus on a typical case in which firm $N$ receives a similar level of exposure in both cases when it does not win the prominent sponsored slot. The parametric assumption is mainly to facilitate a neat comparison and does not affect our qualitative results. We use superscript $C$ to denote the co-listing case and $B$ to denote the benchmark case.

### 4.2 Effect on Social Welfare

We consider the social welfare as the sum of total consumer surplus, both firms’ profits, and the search engine’s revenue. Essentially, social welfare equals the sum of the realized utility of consumers from consuming the products they have purchased. Recall that $M$ always outbids $N$ in equilibrium in the benchmark case and $M$’s equilibrium price is higher than $N$’s in both cases. We therefore can write the equilibrium social welfare as a function of information incompleteness $\psi$ in a uniform expression.

\[
W(\psi, \theta) = \psi \left[ (1-\theta) + \theta \int_{1-p_M^*(\psi, \theta)}^1 p_M(\psi, \theta) \, dx \right] + (1-\psi) \left[ (1-\theta) (1-p_M^*(\psi, \theta) + p_N^*(\psi, \theta)) \right] + (1-\psi) \left[ \theta + (1-\theta) \int_{1-p_M^*(\psi, \theta)}^1 p_M(\psi, \theta) + p_N^*(\psi, \theta) \, dx \right]
\]

(10)

where $p_M^*(\psi, \theta)$ and $p_N^*(\psi, \theta)$ are defined by Eq.(5). The first term represents the total realized utility of those consumers who visit firm $M$’s website only and make purchases there. The second term refers to those who visit both firms’ sites and buy from firm $M$, and the third term refers to those who buy from $N$ after visiting both sites. By definition, the equilibrium social welfare in the benchmark case $W^B = W(\psi_2, \theta)$. The equilibrium welfare in the co-listing case depends on the bidding outcome according to Proposition 1: $W^C = W(\psi_2, \theta)$ when $\theta < \theta^*(\psi_1, \psi_2)$, and $W^C = W(\psi_1, \theta)$ when $\theta > \theta^*(\psi_1, \psi_2)$. As we can see, when $M$ wins the top sponsored slot, the welfare achieved in both cases is the same. What we are interested in is whether the presence of the organic list can increase total social welfare when the niche firm wins the top sponsored slot. The answer is positive.
**Proposition 3.** Organic listing improves the social welfare in that $W^C \geq W^B$ and the strict inequality holds when $\theta > \theta^*(\psi_1, \psi_2)$ (defined by Eq.(7)).

When $\theta > \theta^*(\psi_1, \psi_2)$, $N$ wins the prominent sponsored slot in the co-listing case and increases its exposure rate from $1 - \psi_2$ to $1 - \psi_1$. Proposition 3 reveals that increasing the niche firm’s exposure improves the total welfare. Notice that social welfare achieves its maximum when all consumers purchase their preferred products. From Eq.(10), we can identify two main sources of social efficiency loss: One is informational incompleteness (characterized by the first term in Eq. (10)) and the other is lack of competitiveness (characterized by the second and third terms). When $\psi$ is high, the *informational* efficiency loss is high in the sense that many $N$-type consumers are not aware of product $N$ and end up buying from $M$ instead. Furthermore, a portion of $N$-type consumers who visit only $M$’s website may leave with no purchase because of the high price charged by firm $M$. There is also *competitive* efficiency loss: When the degree of its informational advantage is high, firm $M$ tends to charge a high price. Consequently, a certain portion of marginal $M$-type consumers who visit both sites may purchase product $N$ rather than product $M$, which is socially inefficient. When $\psi$ is reduced, $N$ increases its exposure and $M$ charges a more competitive price in equilibrium, so both informational and competitive efficiency losses are mitigated, which leads to an increase in the overall social welfare.

Different from the benchmark case, in the co-listing case, the free prominence advantage given to the leading firm in the organic list diminishes its sponsored bidding incentive because of the decreased promotive benefit. It thus helps the niche firm to win the prominent sponsored link and better expose itself. In this sense, organic listing adjusts the competence difference among firms by compensating the dominant firm and thus balances the equilibrium informational structure to improve overall social welfare.

### 4.3 Effect on Sales Diversity

We use the *Gini coefficient* to measure the sales diversity. As a popular measure of inequality of income distribution, the Gini coefficient is defined as $G = 1 - 2 \int_0^1 L(x) dx$, where $L(x)$ is the Lorenz curve, which measures the lowest $100x$ percent population’s cumulative income percentage. The Gini coefficient measures the difference between the actual (income) distribution and the perfect equality/diversification case. A higher Gini coefficient indicates a greater degree of inequality, while a
lower coefficient means greater diversification. For discrete cases, the Gini coefficient can be computed as follows,

\[ G = 1 - \frac{1}{n} \sum_{i=1}^{n} \frac{(S_{i-1} + S_{i})}{S_n}, \quad (11) \]

where \( S_i = \sum_{j=1}^{i} y_j \) (and \( S_0 \equiv 0 \)), and \( \{y_i\}_{i=1}^{n} \) is the ordered sequence of the value of interest (e.g., income) for each individual in the population such that \( y_i \leq y_{i+1} \).

In our model, we use the Gini coefficient to measure the diversification of the realized sales across the two firms in equilibrium, and the value of interest in Eq.(11) is the equilibrium sales amount. By substituting the equilibrium sales amount \( D_N (p^*_M(\psi), p^*_N(\psi); \psi) \) and \( D_M (p^*_M(\psi), p^*_N(\psi); \psi) \) derived from Eq.(3) into Eq.(11), we can calculate the Gini coefficient here as follows.

\[ G(\psi) = \frac{1}{2} \frac{D_N (p^*_M(\psi), p^*_N(\psi); \psi)}{D_N (p^*_M(\psi), p^*_N(\psi); \psi) + D_M (p^*_M(\psi), p^*_N(\psi); \psi)} \quad (12) \]

Similarly as before, we have \( G^B = G(\psi_2) \); \( G^C = G(\psi_2) \) when \( \theta < \theta^*(\psi_1, \psi_2) \) and \( G^C = G(\psi_1) \) when \( \theta > \theta^*(\psi_1, \psi_2) \).

**Proposition 4.** Organic listing improves the sales diversity in that \( G^C \leq G^B \) and the strict inequality holds when \( \theta > \theta^*(\psi_1, \psi_2) \) (defined by Eq.(7)).

Similar to the welfare-improving effect, organic listing increases sales diversity when the niche firm outbids the mainstream one. By winning the prominent sponsored position under the co-listing structure, the niche firm increases its exposure and attracts more consumers to visit its site, among whom all the \( N \)-type consumers as well as part of the \( M \)-type consumers purchase from it. Therefore, the realized market share of the niche firm is increased and the overall sales diversity is improved. In this sense, the diversity-improving effect of organic listing and the aforementioned welfare-improving effect share the same origin: adjusting the competence difference among firms by compensating the strong.

### 4.4 Effect on Search Engine Benefit

In evaluating the search engine’s benefit, we consider both the short-term and the long-term aspects. We decompose the search engine benefit, \( SB \), into two parts: immediate revenue, \( IR \), and long-term growth, \( LG \). Because we are interested in the general trends rather than the detailed dynamics, we
model search engine benefit in a general and abstract way such that $SB$ is defined as a function of $IR$ and $LG$, $SB = f(IR, LG)$, where $f(\cdot, \cdot)$ is strictly increasing in both dimensions.

$IR$ is the search engine’s revenue from sponsored bidding when holding the total consumer base constant (and normalized to 1), and $LG$ reflects the change of future customer base. We consider $LG$ as a strictly increasing function of the equilibrium consumer surplus, $CS$, so that $LG = g(CS)$. Here, $CS$ is defined as follows.

\[
CS(\psi, \theta) = \psi \left[ (1 - \theta) (1 - p_M^*(\psi, \theta)) + \theta \int_{p_M^*(\psi, \theta)}^1 (x - p_M^*(\psi, \theta)) \, dx \right] \\
+ (1 - \psi) (1 - \theta) (1 - p_M^*(\psi, \theta) + p_N^*(\psi, \theta)) (1 - p_M^*(\psi, \theta)) \\
+ (1 - \psi) \left[ \theta (1 - p_M^*(\psi, \theta)) + (1 - \theta) \int_{1 - p_M^*(\psi, \theta)}^{1 - p_N^*(\psi, \theta)} (x - p_N^*(\psi, \theta)) \, dx \right],
\]

where $p_M^*(\psi, \theta)$ and $p_N^*(\psi, \theta)$ are equilibrium prices as before. Notice that Eq. (13) equals the expected net utility of any individual customer before entering the search market (without knowing her actual preference before the search). In this sense, we can interpret $g(CS)$ as the future consumer traffic volume attracted to the search engine when consumers have outside options (e.g., competing search engines). More specifically, we may consider that consumers have different reserve utilities which follow a certain distribution, and they use this particular search engine if the expected utility exceeds their reserve values and leave for other options otherwise. As a result, the future consumer traffic volume forms an increasing function of $CS$, as can be represented by $g(CS)$.

In sum, search engine benefit is modeled as

\[
SB = f(IR, LG) = f(IR, g(CS)) = h(IR, CS),
\]

where $\frac{\partial}{\partial IR} h > 0$ and $\frac{\partial}{\partial CS} h > 0$. Notice that the general form of the search engine’s benefit leaves sufficient flexibility so that $SB$ can either be interpreted as the net present value of all future revenue flows in a dynamic context or be considered as incorporating benefits from other non-monetary factors (e.g., reputation and public image).

We are interested in how organic listing affects the search engine’s benefit in both short-term and long-term perspectives. Thus, we next compare the immediate revenue and the long-term growth (i.e., consumer surplus, essentially) under both the benchmark and the co-listing cases.

In second-price auctions, the auctioneer’s revenue equals the second highest bid in equilibrium. In
the benchmark case, since $M$ always wins the auction, search engine’s immediate revenue $IR^B = \hat{b}_N$.

In the case with the organic list, $IR^C = b^*_N$ when $\theta < \theta^* (\psi_1, \psi_2)$ and $IR^C = b^*_M$ when $\theta > \theta^* (\psi_1, \psi_2)$, where $\theta^* (\psi_1, \psi_2)$ is defined by Eq.(7).

**Lemma 2.** Generically, the search engine’s immediate revenue is lower in the co-listing case; that is, when $\frac{5\sqrt{6}}{6} - 2 < \theta < \frac{1}{2}$, $IR^C < IR^B$ for all $0 < \psi_1 < \psi_2 < 1$.

Lemma 2 shows that compared to the case without the organic list, the co-listing case may reduce the search engine’s revenue in the short run. In the spirit of Corollary 1 and Corollary 2, this somewhat disappointing result should not sound too surprising. It underscores the fact that the free prominence advantage given to the leading firm in the organic list may reduce advertisers’ bidding incentives and hence sacrifice the search engine’s direct revenue.

A brief reasoning for the above result is as follows. Since $IR^B = \hat{b}_N$ and $IR^C = \min \{b^*_M, b^*_N\}$, to conclude that $IR^B > IR^C$, it is sufficient to show that $\hat{b}_N > b^*_N$. Recall that a firm’s equilibrium bid equals the difference between its equilibrium profits when winning the top sponsored slot and when not winning it (i.e., according to Tables 1 and 2, $b^*_N = \pi^*_N (\psi_1, \theta) - \pi^*_N (\psi_2, \theta)$ and $\hat{b}_N = \hat{\pi}^1_N - \hat{\pi}^2_N$). On the one hand, as discussed earlier, when $N$ does not win the top sponsored slot, the equilibrium profit achieved in both cases is the same (i.e., $\pi^*_N (\psi_2, \theta) = \hat{\pi}^2_N$). On the other hand, when $N$ wins the top sponsored slot, in the co-listing case, $M$ still possesses significant prominence from the organic list, which limits $N$’s profit due to the consequent intense price competition; in the benchmark case, however, $N$ could overturn the informational dominance structure thoroughly and greatly improve the profitability of winning the top sponsored slot. As a result, $N$’s winning profit in the benchmark case is higher in general (i.e., $\hat{\pi}^1_N > \pi^*_N (\psi_1, \theta)$). In consequence, as long as $\hat{b}_N > 0$ (i.e., under some boundary condition on $\theta$ according to Corollary 2), we can conclude that $\hat{b}_N > b^*_N$ and hence $IR^B > IR^C$.

It is also worth noting that, in addition to $N$’s decreased bidding incentive, firm $M$’s incentive to bid for a sponsored link is also less under the co-listing case than under the benchmark case, sometimes even to a greater degree such that firm $M$ might lose the bidding competition to firm $N$ in the co-listing case. In other words, paralleling the organic list with the sponsored list generally results in lower bidding incentives for both firms.

The effect on the equilibrium consumer surplus can be analyzed in a similar fashion as the social welfare and the sales diversity. By Eq.(13), $CS^B = CS (\psi_2, \theta)$ and $CS^C = CS (\psi_1, \theta)$ when $\theta >$
\( \theta^* (\psi_1, \psi_2) \).

**Lemma 3.** The equilibrium consumer surplus in the co-listing case is higher in that \( CS^C \geq CS^B \) and the strict inequality holds when \( \theta > \theta^* (\psi_1, \psi_2) \) (defined by Eq.(7)).

The above result can be understood based on the same two sources of loss in analyzing the equilibrium social welfare: informational and competitive. In the co-listing case, the diluted bidding incentive of the leading firm gives the niche firm more chances to win a better sponsored position for a better exposure. As a result, consumers are more likely to find the product they prefer—at lower prices due to the intensified price competition. Therefore, by inducing a lower level of informational incompleteness \( \psi \) in equilibrium, the co-listing case improves the overall consumer surplus.

It is worth noting that the co-listing structure manages to induce the niche firm to increase its exposure in the sponsored list while keeping the organic list “organic.” Therefore, it increases the surplus of the potential consumers (i.e., those who are looking for product information) and meanwhile ensures the general search engine users’ utilities. Such effects promise growth of the user base in the long run.

Combining the results from Lemma 2 and Lemma 3, we can conclude that the effect of organic listing on the search engine’s benefit is a balance between short-term profitability and long-term growth.

**Proposition 5.** Organic listing trades off the short-term revenue for the long-term benefit of the search engine in that \( IR^C < IR^B \) and \( LG^C > LG^B \) when \( \theta > \max\{\theta^* (\psi_1, \psi_2), \frac{5\sqrt{5}}{6} - 2\} \).

An immediate result from Proposition 5 is that if the search engine values the long-term benefit enough (i.e., if \( \frac{\partial}{\partial LG} f \gg \frac{\partial}{\partial IR} f \) as in Eq.(14)), then the overall search engine benefit can be much higher in the co-listing case. In this sense, separating the organic list out as an independent major list is essentially a choice between myopic and long-sighted perspectives.

To conclude, this section analyzes the effects of organic listing on the equilibrium outcomes. Overall, organic listing reduces search engine revenue in the short run but improves the equilibrium social welfare, sales diversity, and consumer surplus, which eventually benefits the search engine in the long run.

It is worth pointing out that the aforementioned beneficial effects of organic listing function only under moderate levels of market asymmetry (i.e., firms are not too different in market preference
so $\theta$ is not too small), which can be considered as the general cases for most markets in reality. It hence justifies the design of organic listing in general. Nevertheless, these effects malfunction when the competence difference among advertisers is too large to adjust. For firms at the very tail end of the distribution (i.e., for niche firms with very small $\theta$), their bidding incentive is still lower than that of their strong competitors. As a result, the leading firms dominate the prominence in both lists and no fundamental change occurs in information structure after the sponsored bidding competition. Consequently, while bearing revenue loss, the search engine fails to engender structural improvement in social welfare, sales diversity, as well as consumer surplus. In this sense, the typical organic listing design serves the “middles” well, rather than the “tails.”

5 Improving Organic Ranking

The previous section discusses the beneficial effects of organic listing and also pinpoints the malfunction of such effects with regard to “tail” firms. In this section, we propose a possible direction that could mitigate such drawbacks and can better serve the tails.

Instead of assuming that the organic list ranks firms in an order based strictly on their relative popularity (which we refer to as pure organic ranking mechanism), we consider a mixed organic ranking mechanism in which the organic list ranks firms by their popularity with certain probability and in an inverse order otherwise. Following the model setup in Section 2, a mixed organic ranking mechanism can be characterized by a randomization parameter $\lambda$ ($0 \leq \lambda \leq 1$) such that with probability $1 - \lambda$ the mainstream firm is listed in a prominent organic slot $i_M$ on the SERP and the niche firm is listed in a less-prominent organic slot $i_N$; meanwhile, with probability $\lambda$, firm $N$ is given a prominent organic position $i'_N$ and $M$ stays in a less-prominent organic position $i'_M$. Notice that $\alpha_{i_M} > \alpha_{i_N}$, $\alpha'_{i'_N} > \alpha'_{i'_M}$, and the pure organic ranking is thus a special case of mixed ranking with $\lambda = 0$.

To be consistent with the baseline model, we let $\alpha'_{i'_N} = \alpha_{i_M} = 1$ to model the significant attention-catching effect of a prominent organic position. When firm $M$ wins the first sponsored slot, the probability of $M$’s being visited by a consumer ($q^1_M$) and the probability of $N$’s being visited ($q^2_N$) are

\begin{align*}
q^1_M &= (1 - \lambda) + \lambda \left[ 1 - (1 - \alpha_{i'_M})(1 - \beta_1)(1 - \gamma_{i'_M1}) \right] \\
q^2_N &= (1 - \lambda) \left[ 1 - (1 - \alpha_{i_N})(1 - \beta_2)(1 - \gamma_{i_N2}) \right] + \lambda
\end{align*}

(15)
Similarly, when firm N wins the first sponsored slot, the probability of N’s being visited \( (q_N^1) \) and that of M’s being visited \( (q_M^2) \) are

\[
\begin{align*}
q_M^2 &= (1 - \lambda) + \lambda \left[ 1 - (1 - \alpha_i'')(1 - \beta_2)(1 - \gamma_i'') \right] \\
q_N^1 &= (1 - \lambda) \left[ 1 - (1 - \alpha_i')(1 - \beta_1)(1 - \gamma_i') \right] + \lambda
\end{align*}
\]

(16)

To simplify the discussion, we let \( (1 - \alpha_i')(1 - \beta_1)(1 - \gamma_i) = (1 - \alpha_i')(1 - \beta_1)(1 - \gamma_i) \equiv \psi_1 \) and \( (1 - \alpha_i')(1 - \beta_2)(1 - \gamma_i) = (1 - \alpha_i')(1 - \beta_2)(1 - \gamma_i) \equiv \psi_2, \psi_1 < \psi_2 \). A mixed organic ranking mechanism that randomly switches M’s and N’s positions with probability \( \lambda \) is an example satisfying the above equality. For simplicity, here we let \( \psi_1 = 0 \) because the first sponsored link often attracts significant attention so that \( (1 - \beta_1) \) can be sufficiently small. The parametric assumptions are only to facilitate derivation of neat analytical results and are not essential for the qualitative results to hold. As we can show numerically, insights remain the same when \( \psi_1 \) is extended to be positive.

The underlying idea of the mixed organic ranking mechanism is to strategically introduce uncertainty in firms’ organic ranks so that firms’ relative prominence advantage can be reversed probabilistically. In practice, mixed ranking can be implemented by simply randomizing the ordering of targeted advertisers with the desired probability. It can also be interpreted as including additional factors other than the popularity measure into the organic ranking rule.

We can formulate firms’ demand functions in a similar way. When firm M wins the first sponsored slot, M possesses informational monopoly power if firm N is placed in the less prominent organic slot (with probability \( 1 - \lambda \)) and is not noticed by a consumer (with probability \( \psi_2 \)). Therefore, the demand functions facing both firms are as follows.

\[
\begin{align*}
D_M^1 (p_M, p_N) &= (1 - \lambda) \psi_2 A_M (p_M) + \left[ 1 - (1 - \lambda) \psi_2 \right] S_M (p_M, p_N) \\
D_M^2 (p_N, p_M) &= \left[ 1 - (1 - \lambda) \psi_2 \right] S_M (p_N, p_M)
\end{align*}
\]

where \( A_i (\cdot) \) and \( S_i (\cdot) \) are defined as before by Eq.(1) and Eq.(2). When firm N wins the sponsored bidding, similarly, the demand functions become

\[
\begin{align*}
D_M^2 (p_M, p_N) &= (1 - \lambda \psi_2) S_M (p_M, p_N) \\
D_N^1 (p_N, p_M) &= \lambda \psi_2 A_N (p_N) + \left[ 1 - \lambda \psi_2 \right] S_N (p_N, p_M)
\end{align*}
\]

(17)

(18)

Comparing the above with Eq.(3), we can see the major difference from the original model with pure
organic ranking. By strategically randomizing the organic ranking and probabilistically altering the relative prominence advantage in the organic list, the mainstream firm no longer possesses guaranteed prominence advantage. In particular, if it fails to win the sponsored bidding, the mainstream firm might even become inferior to its competitor in terms of informational exposure. Hence, it is now possible for the niche firm to gain a certain level of prominence dominance by winning the prominent sponsored position.

Along a similar approach as before, by considering the best response to its competitor’s price in the second stage, we can first derive firms’ equilibrium price conditional on whether they acquire the first or the second sponsored position, \( \tilde{p}_i^j \), where \( \{i, j\} = \{M, N\} \) and \( \{j, \tilde{j}\} = \{1, 2\} \). In other words, \( \tilde{p}_i^j \) solves the profit maximization problem \( \max_{p} p D_i^j (p, \tilde{p}_i^j) \). Plugging \( \{\tilde{p}_i^j, \tilde{p}_i^j\} \) back into the profit function, we can derive the second-stage equilibrium profits for firms in both cases: \( \tilde{\pi}_i^j = \tilde{p}_i^j D_i^j (\tilde{p}_i^j, \tilde{p}_i^j) \). Back to the first-stage bidding competition, again, bidding the true value \( \Delta \tilde{\pi}_i = \tilde{\pi}_i^1 - \tilde{\pi}_i^2 \) is the unique perfect equilibrium, that is, \( \tilde{b}_i = [\Delta \tilde{\pi}_i]^+, i \in \{M, N\} \).

We are interested to see whether introducing mixed organic ranking can improve the equilibrium outcome, especially when \( \theta \) is small. In particular, we investigate the marginal effect of increasing the randomizing parameter \( \lambda \) on the equilibrium welfare, sales diversity, and both the short-term and the long-term benefit of the search engine, evaluated at \( \lambda = 0 \) (which is the case of pure organic ranking). Notice that when \( \lambda = 0 \), the equilibrium outcome is the same as in the original model: When \( \theta < \theta^* (0, \psi) \) (defined by Eq. (7)), we have \( \tilde{b}_M > \tilde{b}_N \) and \( \tilde{p}_M > \tilde{p}_N \). As a result, the equilibrium revenue for the search engine equals \( \tilde{IR} (\psi_2, \theta, \lambda) = \Delta \tilde{\pi}_N (\psi_2, \theta, \lambda) \). The equilibrium consumer surplus and social welfare can be written as

\[
\tilde{CS} (\psi_2, \theta, \lambda) = (1 - \lambda) \psi_2 \left[ (1 - \theta) (1 - \tilde{p}_M^1) + \theta \int_{\tilde{p}_M^1}^{1} (x - \tilde{p}_M^1) dx \right] + [1 - (1 - \lambda) \psi_2] (1 - \theta) (1 - \tilde{p}_M^1 + \tilde{p}_N^2) (1 - \tilde{p}_M^1) + [1 - (1 - \lambda) \psi_2] \left[ \theta (1 - \tilde{p}_N^2) + (1 - \theta) \int_{1 - \tilde{p}_M^1 + \tilde{p}_N^2}^{1} (x - \tilde{p}_N^2) dx \right],
\]

and

\[
\tilde{W} (\psi_2, \theta, \lambda) = (1 - \lambda) \psi_2 \left[ (1 - \theta) + \theta \int_{\tilde{p}_M^1}^{1} x \, dx \right] + [1 - (1 - \lambda) \psi_2] (1 - \theta) (1 - \tilde{p}_M^1 + \tilde{p}_N^2) + [1 - (1 - \lambda) \psi_2] \left[ \theta + (1 - \theta) \int_{1 - \tilde{p}_M^1 + \tilde{p}_N^2}^{1} x \, dx \right].
\]

The sales Gini coefficient in equilibrium is

\[
\tilde{G} (\psi_2, \theta, \lambda) = \frac{1}{2} - \frac{D_N^2 (\tilde{p}_N^2, \tilde{p}_M^1)}{D_M^1 (\tilde{p}_M^1, \tilde{p}_N^2) + D_N^2 (\tilde{p}_N^2, \tilde{p}_M^1)}. \tag{21}
\]
Proposition 6. When $\theta < \theta_0$ and $\psi_2 < \frac{1}{3(1-\lambda)}$, introducing mixed organic ranking can improve the search engine’s immediate revenue and long-term growth, social welfare, and sales diversity concurrently in that (i) $\frac{\partial}{\partial \lambda} \Delta \tilde{\pi}_N (\psi_2, \theta, \lambda) |_{\lambda=0} > 0$; (ii) $\frac{\partial}{\partial \lambda} \tilde{G}S (\psi_2, \theta, \lambda) |_{\lambda=0} > 0$ (iii) $\frac{\partial}{\partial \lambda} \tilde{W} (\psi_2, \theta, \lambda) |_{\lambda=0} > 0$; and (iv) $\frac{\partial}{\partial \lambda} \tilde{G} (\psi_2, \theta, \lambda) |_{\lambda=0} < 0$. Here, $\theta_0 \in (0, \frac{1}{2})$ solves the equation $\frac{\partial}{\partial \lambda} \Delta \tilde{\pi}_N (\psi_2, \theta_0, \lambda) |_{\lambda=0} = 0$.

The proposition indicates that mixed organic ranking improves both profitability and efficiency in the cases when pure ranking is unable to engender satisfactory revenue, efficiency, and diversity. Recall the results from Corollary 1 and Propositions 3 through 5: When $\theta$ is small and $\psi_2$ is not too large, pure organic ranking cannot induce structural improvement in consumer surplus, social welfare, or sales diversity, while search engine revenue is extremely low. In this sense, Proposition 6 shows that introducing randomization serves as a good remedy to relieve the major drawback of the pure organic ranking mechanism.

Under pure organic ranking, as is discussed, when $\theta$ is small and $\psi_2$ is not too large, the niche firm’s promotive incentive for winning a prominent sponsored position completely vanishes due to its huge disadvantage in market preference and the mainstream firm’s unshakable prominence dominance in organic listing, which hurts the revenue contributed to the search engine. In contrast, mixed organic ranking gives the niche firm chances to occupy the prominent position in the organic list, which makes it profitable to win the prominent sponsored slot because the niche firm can exploit informational monopoly in these cases. In other words, introducing mixed ranking adds preventive incentive to the niche firm’s sponsored bidding motivation. Essentially, as bidders become more comparable, they are induced to bid more aggressively, and thus a properly set randomization factor can improve the auctioneer’s revenue. This rationale is along the same spirit of promoting disadvantageous players or handicapping advantageous players for competition purposes in existing studies (Liu et al., 2010).

On the other hand, the equilibrium consumer surplus and social welfare are improved as well. The reason is that the mixed ranking reduces the aforementioned two sources of efficiency loss. Occasional perturbation in organic rank directly improves firm $N$’s exposure and thus effectively reduces the informational loss of consumer surplus and social welfare. Meanwhile, a smaller prominence difference induces more intense price competition so that the competitive efficiency loss is also well controlled. Given the increase in the surplus of the potential consumers and considering that slight perturbation of commercial websites’ links for certain markets would have little impact on general search engine
users (who are mainly interested in non-commercial websites), overall, introducing mixed organic ranking could benefit the search engine in the long run.

An interesting aspect of Proposition 6 is that welfare and revenue can be improved simultaneously so that the search engine’s short-term and long-term interests are aligned, unlike most existing studies in which increasing revenue is often at the cost of welfare. The key to this result is the auto-balance between organic ranking and sponsored bidding. Although mixed ranking reduces the mainstream firm’s organic exposure, it does not significantly decrease the mainstream firm’s overall exposure because the mainstream firm regains essential prominence by winning the prominent sponsored position. Therefore, mixed ranking promotes the weak player’s prominence at little cost to the stronger one’s exposure, which leads to less total informational efficiency loss and thus higher consumer surplus and social welfare. In this sense, a mixed organic ranking mechanism allocates the total resource of consumer attention in a more effective way.

Previous discussion shows that it is hardly possible for small firms facing highly asymmetric market preference to prevail in terms of equilibrium sales amount. In fact, the organic list in the co-listing case (with pure ranking) performs as an implicit adjustment to promote the weak players, which has been shown to serve the moderately-weak better than the very-tail. To mitigate this flaw and better serve the tails, the mixed organic ranking explicitly adjusts the competence difference by directly promoting the weak in the organic list and effectively increasing its exposure. As a result, equilibrium sales diversity improves (the Gini coefficient decreases), and even the weak firms with very small $\theta$ manage to achieve higher market share than under pure ranking.

6 Conclusion

Studying the intriguing role of organic listing in search advertising, we take a different perspective by focusing on the effects of organic listing as a competing information source on advertisers’ advertising strategies as well as the equilibrium outcomes. This paper thus complements the existing literature in deepening the understanding of this issue. This paper also provides implications for search engine designers and marketing managers.

First, we provide economic justification for the common practice of the co-listing structure (with both organic list and sponsored links) in the search industry. On the surface, organic lists provide
potential advertisers with prominence for free, which reduces their bidding incentive for sponsored exposure and thus hurts search engine revenue. Nevertheless, as leading firms typically gain more free prominence and their bidding incentive would decline more than less competitive firms, organic listings could leverage the competence difference and help the less competitive advertisers win a better position in the sponsored lists, which increases information completeness and improves equilibrium consumer surplus, social welfare and sales diversity. In this sense, organic listing may sacrifice profitability from a myopic view, but it could accelerate the growth of the user base and result in long-term benefit.

Furthermore, we underscore the importance of keeping organic listing “organic”: Organic ranking should be strictly protected from advertisers’ manipulation. Throughout this paper, we take a cautious approach that organic listing stays under full control of the search engine. Nevertheless, two alternatives sometimes seem appealing to different parties and can occasionally be observed in the industry: Search engines may be tempted to “sell” slots in organic listing, or a third party may be interested in offering the service of promoting firms’ organic ranking. As we show, the beneficial effects of the co-listing structure root in that it improves the equilibrium information structure through adjusting advertisers’ sponsored bidding incentives rather than impairing the objectivity of the organic listing. Selling organic positions violates this spirit. It may increase the search engine’s short-term revenue, but would eventually hurt the long-term growth. This could be the reason why the leading search engine in China, Baidu, has received increasing criticism for its selling of top organic slots to paid advertisers (Webster, 2008). The second alternative, known as search engine optimization (SEO), may cause similar problems; meanwhile, advertisers’ expenditure flows to SEO companies, which would be even worse from search engines’ perspective. It explains the practice that all search engines keep their organic ranking mechanism private and constantly develop more sophisticated mechanisms by introducing new factors into the ranking rule so as to adapt to possible SEO manipulation.

Meanwhile, we also suggest that while keeping organic listing unaltered from advertisers, the organic ranking mechanism could be further improved under certain cases. As we pinpoint, the drawbacks of popularity-based organic ranking arise in the presence of highly asymmetric market preference. We propose that slightly mixing the organic ranks of targeted commercial websites probabilistically may help improve the equilibrium outcomes for those markets, increasing direct revenue,
consumer surplus, welfare, and diversity at the same time. This possible direction calls for novel algorithm designs to incorporate strategic perturbation into the organic ranking methods. Possible implementations might include randomizing between regular and reverse ordering of targeted advertisers with certain probability, or introducing additional factors beyond website relevance into the ranking score.

In addition to the implications for search engine designers, our study also provides rationale for marketing managers in dealing with online search advertising. Advertisers whose websites have already been placed in prominent positions in organic lists may wonder whether it is still worth investing in sponsored bidding. We suggest that sponsored bidding is rewarding when they have salient advantage in market preference, and dominating exposure in both lists maximizes their profitability. When market preference is relatively diversified, advertisers whose organic position is not satisfactory should bid aggressively to win the top sponsored slots because the marginal benefit of increasing sponsored exposure is fairly high in this case. In contrast, for very niche firms or individual sellers without sufficient competence in market preference, although it may seem appealing to expose themselves via this new advertising medium, excessive spending in sponsored bidding may have a low return on investment, because even a very prominent sponsored position with high click-through rate may bring high advertising bills but not commensurable profits.

This paper triggers interesting directions for future research. For example, as we suggest that organic ranking could be further improved, especially for highly asymmetric markets, it leads to a challenging yet important research direction: the optimal design of organic ranking mechanism, taking into consideration advertisers’ reaction in both sponsored bidding competition and product market competition. Notice that the optimal organic ranking mechanism should depend on the competitive situation among advertisers, which brings challenges for design science to devise novel algorithms to implement such situation-dependent dynamic ranking of organic links.

References


A Appendix

Due to the page limit, we provide the outline of all proofs only. Detailed proofs are available upon request.

Proof of Lemma 1. According to Table 1, (i) If $\frac{1}{3} \leq \psi < 1$, then $\frac{\partial^2}{\partial \psi \partial \theta} \pi_N^* (\psi, \theta) = -1 < 0$ and $\frac{\partial^2}{\partial \psi \partial \theta} \pi_M^* (\psi, \theta) = -\frac{1}{4(1-\theta)^2} < 0$.

(ii) If $0 < \psi < \frac{1}{3}$, then $\frac{\partial^2}{\partial \psi \partial \theta} \pi_M^* (\psi, \theta) = \frac{f(\psi, \theta)}{(3-\theta-3\psi+7\theta\psi)^2}$, where $f(\psi, \theta)$ is a polynomial function of $\psi$ and $\theta$ with degree of $8(\psi^4 \theta^4)$. We can show that $f(\psi, \theta)$ is convex in $\psi$ for $0 < \psi < \frac{1}{3}$ and $0 < \theta < \frac{1}{2}$. We can verify that $f(0, \theta) < 0$ and $f(\frac{1}{3}, \theta) < 0$ for all $\theta \in (0, \frac{1}{2})$. Therefore, $f(\psi, \theta) < 0$ for all $\theta \in (0, \frac{1}{3})$ and $\forall \theta \in (0, \frac{1}{2})$, which implies $\frac{\partial^2}{\partial \psi \partial \theta} \pi_M^* (\psi, \theta) < 0$. Similarly, $\frac{\partial^2}{\partial \psi \partial \theta} \pi_N^* (\psi, \theta) = \frac{g(\psi, \theta)}{(1-\theta)^2(3-\theta-3\psi+7\theta\psi)^2}$, where $g(\psi, \theta)$ is a polynomial function of $\psi$ and $\theta$ with degree of $10(\psi^4 \theta^6)$. Again, we can show that $g(\psi, \theta)$ is convex in $\psi$ for $0 < \psi < \frac{1}{3}$ and $0 < \theta < \frac{1}{2}$. Since $g(0, \theta) < 0$ and $g(\frac{1}{3}, \theta) < 0$ for all $\theta \in (0, \frac{1}{2})$, we can conclude that $g(\psi, \theta) < 0$ for all $\theta \in (0, \frac{1}{3})$ and $\forall \theta \in (0, \frac{1}{2})$. Therefore, $\frac{\partial^2}{\partial \psi \partial \theta} \pi_N^* (\psi, \theta) < 0$.

Proof of Proposition 1. Define $h(\psi, \theta) = \pi_M^* (\psi, \theta) + \pi_N^* (\psi, \theta)$. Since both $\pi_M^* (\psi, \theta)$ and $\pi_N^* (\psi, \theta)$ are submodular by Lemma 1, $h(\psi, \theta)$ is also submodular. We then define

$$H(\theta, \psi_1, \psi_2) \equiv \left[ \pi_M^* (\psi_2, \theta) - \pi_M^* (\psi_1, \theta) \right] - \left[ \pi_N^* (\psi_1, \theta) - \pi_N^* (\psi_2, \theta) \right] = h(\psi_2, \theta) - h(\psi_1, \theta).$$

If $0 < \psi_1 < \psi_2 < \frac{1}{3}$ or $\frac{1}{3} \leq \psi_1 < \psi_2 < 1$, then

$$\frac{\partial}{\partial \theta} H(\theta, \psi_1, \psi_2) = \frac{\partial}{\partial \theta} h(\psi_2, \theta) - \frac{\partial}{\partial \theta} h(\psi_1, \theta) = \frac{\partial^2}{\partial \theta \partial \psi} h(\psi, \theta)(\psi_2 - \psi_1)$$

where the second equality is by applying the Mean Value Theorem and $\bar{\psi} \in [\psi_1, \psi_2]$. Since $\frac{\partial^2}{\partial \theta \partial \psi} h(\psi, \theta) <
Since \( \Delta \hat{\psi}^0 \) is a polynomial function of \( \psi \) and \( \theta \), we let

\[
H(\theta, \psi_1, \psi_2) = h(\psi_2, \theta) - h(\psi_1, \theta) = \int_0^\frac{1}{3} \frac{\partial^2}{\partial \theta \partial \psi} h(\psi, \theta) \, d\psi + \int_{\frac{1}{3}}^{\frac{1}{3}} \frac{\partial^2}{\partial \theta \partial \psi} h(\psi, \theta) \, d\psi.
\]

Since \( \frac{\partial^2}{\partial \theta \partial \psi} h(\psi, \theta) < 0 \), \( \frac{\partial}{\partial \theta} H(\theta, \psi_1, \psi_2) < 0 \).

If \( 0 < \psi_1 < \frac{1}{3} \leq \psi_2 < 1 \), then

\[
\frac{\partial}{\partial \theta} H(\theta, \psi_1, \psi_2) = \frac{5(\psi_2 - \psi_1)}{9(1 - \psi_1)(1 - \psi_2)} \text{ (in the case of 0 < } \psi_1 < \psi_2 < \frac{1}{3} \text{)}, \text{ or } \frac{\psi_2 - \psi_1}{4} \text{ (in the case of } \frac{1}{3} \leq \psi_1 < \psi_2 < 1 \text{)}, \text{ or } \frac{20 - 9(1 - \psi_2)(3 + \psi_1)}{36(1 - \psi_2)} \text{ (in the case of } 0 < \psi_1 < \frac{1}{3} \leq \psi_2 < 1 \text{).}
\]

We can verify that \( H(0, \psi_1, \psi_2) > 0 \) in all three cases. Similarly, \( H\left(\frac{1}{3}, \psi_1, \psi_2\right) = -\frac{\psi_2 - \psi_1}{2} < 0 \) in all three cases. Since \( H(\theta, \psi_1, \psi_2) \) is continuous in \( \theta \), we can conclude that there exists a cutoff \( \theta^* \in (0, \frac{1}{3}) \) such that \( H(\theta^*, \psi_1, \psi_2) = 0 \), \( H(\theta, \psi_1, \psi_2) > 0 \) for \( \theta < \theta^* \), and \( H(\theta, \psi_1, \psi_2) < 0 \) for \( \theta > \theta^* \). Notice that \( \pi^*_M(\psi_2, \theta) - \pi^*_M(\psi_1, \theta) > 0 \) always holds, that is, \( b^*_M > 0 \).

Therefore, we can conclude that \( b^*_M > b^*_N \) for \( \theta < \theta^* \), and \( b^*_M < b^*_N \) for \( \theta > \theta^* \).

**Proof of Corollary 1.** According to Table 1, when \( \psi < \frac{1}{3} \), we can conclude \( \Delta \hat{\Lambda}_i = \Delta \hat{\pi}_i = \Delta \hat{\pi}^2_i, i \in \{M, N\} \). When \( \frac{1}{3} \leq \psi < 1 \), according to Table 2, \( \Delta \hat{\Lambda}_M - \Delta \hat{\Lambda}_N = \frac{(1 - \theta^*)}{4(3 + \psi^2)(1 - \theta^*)} F(\psi, \theta) \), where

\[
F(\psi, \theta) = -4(\psi^2 + 3\psi^2 + 5\psi - 1) \theta + (5\psi^2 + 17\psi^2 + 19\psi + 7).
\]

Since \( F(\psi, 0) > 0 \), \( F(\psi, \frac{1}{2}) > 0 \), and \( F(\psi, \theta) \) is linear in \( \theta \), we can conclude that \( F(\psi, \theta) > 0 \) and hence \( \Delta \hat{\Lambda}_M - \Delta \hat{\Lambda}_N > 0 \) for \( \forall \theta \in (0, \frac{1}{3}) \).

When \( 0 < \psi < \frac{1}{3} \), similarly, \( \Delta \hat{\Lambda}_M - \Delta \hat{\Lambda}_N = \frac{\psi(1 - \theta^*)}{(1 - \theta)^3(3 + \psi^2 - 3\psi + 7\psi)^2} G(\psi, \theta) \), where \( G(\psi, \theta) \) is a polynomial function of \( \psi \) and \( \theta \) with degree of 7 (\( \psi^4 \theta^3 \)). As it can be verified that \( G(\psi, \frac{1}{2}) > 0 \) and \( \frac{\partial}{\partial \theta} G(\psi, \theta) < 0 \) for \( \forall \theta \in (0, \frac{1}{2}) \), \( G(\psi, \theta) > 0 \) and thus \( \Delta \hat{\Lambda}_M - \Delta \hat{\Lambda}_N > 0 \) for \( \forall \theta \in (0, \frac{1}{2}) \).

We can further check that \( \hat{b}_M = [\Delta \hat{\Lambda}_M]^+ > 0 \). Therefore, \( \hat{b}_M > \hat{b}_N \) for \( \forall \psi \in (0, 1) \) and \( \forall \theta \in (0, \frac{1}{2}) \).
Proof of Corollary 2. (i) When $\psi < \frac{1}{3}$, according to Table 2, $\hat{\pi}_N^1 - \hat{\pi}_N^2$ can be simplified as
$$\psi F(\psi, \theta) F(\psi, \theta) \frac{\psi}{(1-\theta)(3(1-\theta)(1-\psi)+4\theta \psi)},$$
where $F(\psi, \theta)$ is a polynomial function of $\psi$ and $\theta$ with degree of $8$ ($\theta^4 \psi^3$). We can show that $\frac{\partial^2}{\partial \psi^2} F(\psi, \theta)$ is monotonic in $\theta$ for $\forall \theta \in (0, \frac{1}{2})$, and both $\frac{\partial^2}{\partial \psi^2} F(\psi, 0)$ and $\frac{\partial^2}{\partial \psi^2} F(\psi, \frac{1}{2})$ are negative. Therefore, $\frac{\partial^2}{\partial \psi^2} F(\psi, \theta) < 0$, and thus $\frac{\partial}{\partial \psi} F(\psi, \theta)$ is decreasing in $\theta$. We can further verify that $\frac{\partial}{\partial \psi} F(\psi, \frac{1}{2}) > 0$ and thus $\frac{\partial}{\partial \psi} F(\psi, \theta) > 0$ for $\forall \theta \in (0, \frac{1}{2})$. We also notice that $F\left(\frac{1}{5}, \frac{5}{6} \pi - 2\right) = 0$ and $F(\psi, \frac{5}{6} \pi - 2)$ is decreasing in $\psi$, which leads to $F\left(\hat{\psi}, \frac{5}{6} \pi - 2\right) > 0$ for $\forall \psi \in (0, \frac{1}{2})$. Therefore, $F(\psi, \theta) > 0$ and $\hat{\pi}_N^1 > \hat{\pi}_N^2$ for $\forall \theta \in (\frac{5}{6} \pi - 2, \frac{1}{2})$ and $\forall \psi \in (0, \frac{1}{2})$.

(ii) When $\psi \geq \frac{1}{3}$, $\pi_N^1$ is decreasing in $\psi$. If we view $\pi_N^1$ and $\pi_N^2$ as functions of $\psi$, it is sufficient to show $\hat{\pi}_N^1(\psi) > \hat{\pi}_N^2(\frac{1}{3})$ for $\forall \psi \geq \frac{1}{3}$. Notice that $\pi_N^1(\psi) - \pi_N^2(\frac{1}{3}) = \frac{G(\psi, \theta)}{6(1-\theta)(3(1-\theta)(1-\psi)+4\theta \psi)},$ where $G(\psi, \theta) = 6(1-\psi)^2 \theta^2 + 12(1-\psi^2) \theta + (\theta^2 + 6 \psi - 3)$. Since $\frac{\partial G}{\partial \psi} > 0$, $\frac{\partial G}{\partial \theta} > 0$, and $G\left(\hat{\psi}, \frac{5}{6} \pi - 2\right) = 0$, $G(\psi, \theta) > 0$ and thus $\hat{\pi}_N^1(\psi) > \hat{\pi}_N^2(\frac{1}{3})$ for $\forall \psi \in (\frac{1}{3}, 1)$ and $\forall \theta \in (\frac{5}{6} \pi - 2, \frac{1}{2})$.

Proof of Proposition 3. We plug in $p_M^*(\psi, \theta)$ and $p_N^*(\psi, \theta)$ from Eq.(5) into Eq.(10), and want to show $\frac{\partial}{\partial \psi} W(\psi, \theta) < 0$.

(i) When $\psi < \frac{1}{3}$, $\frac{\partial}{\partial \psi} W(\psi, \theta) = \frac{F(\psi, \theta)}{2(1-\theta)(3(1-\theta)(1-\psi)+4\theta \psi)},$ where $F(\psi, \theta)$ is a polynomial function of $\psi$ and $\theta$ with degree of $8$ ($\theta^5 \psi^3$). By its concavity ($\frac{\partial^2}{\partial \psi^2} F(\psi, \theta) < 0$), we can show that $F(\psi, \theta) < 0$ for $\forall \psi \in (0, 1)$. Therefore, $\frac{\partial}{\partial \psi} W(\psi, \theta) < 0$, for $\forall \psi \in (0, \frac{1}{3})$ and $\forall \theta \in (0, \frac{1}{2})$.

(ii) When $\psi \geq \frac{1}{3}$, $\frac{\partial}{\partial \psi} W(\psi, \theta) = \frac{12(2-\theta)-12(1-\theta)}{8(1-\theta)}$, which is negative when $\theta > \frac{1}{2} - \frac{\sqrt{6}}{6}$. When $\theta < \frac{1}{2} - \frac{\sqrt{6}}{6}$, we can check that $M$ outbids $N$ in both cases, and therefore $W^B = W^C$.

Altogether, according to Propositions 1 and 2, when $\theta > \theta^*(\psi_1, \psi_2)$, $W^C = W(\psi_1, \theta) > W(\psi_2, \theta) = W^B$; when $\theta < \theta^*(\psi_1, \psi_2)$, $W^C = W^B = W(\psi_2, \theta)$.

Proof of Proposition 4. (i) When $\psi < \frac{1}{3}$, $D_M(\psi) = \frac{[(2-\theta)^2+3(2-\theta)(\theta-1)]^2}{3(1-\theta)(1-\psi)+4\theta \psi}$ and $D_N(\psi) = \frac{[(1+\theta)(1-\theta)+\theta(2-\theta)\psi]}{3(1-\theta)(1-\psi)+4\theta \psi}$. By Eq.(12), we have $\frac{\partial}{\partial \psi} G(\psi) = \frac{\theta \cdot F(\psi, \theta)}{[-3+3\theta+3(1-\theta)^2(1-\psi)+4\theta \psi ]^2},$ where $F(\psi, \theta)$ is a quadratic function of $\psi$ and can be proved positive. (If we write $F(\psi, \theta)$ as $A\psi^2 + B\psi + C$, we can show $A > 0$ and $B^2 - 4AC < 0$.) Therefore, $\frac{\partial}{\partial \psi} G(\psi) > 0$, for $\forall \psi \in (0, \frac{1}{3})$ and $\forall \theta \in (0, \frac{1}{2})$. (ii) When $\psi \geq \frac{1}{3}$, $D_M(\psi) = \left(\frac{1}{2} - \theta\right)\psi + \frac{1}{2}$ and $D_N(\psi) = \frac{1}{2} - \psi$. Therefore, $G(\psi) = \frac{1}{2(1-\theta)}$, which is increasing in $\psi$. Altogether, $\frac{\partial}{\partial \psi} G(\psi) > 0$ for $\forall \theta \in (0, \frac{1}{2})$ and thus, according to Propositions 1 and 2, $G^C = G(\psi_1) < G(\psi_2) = G^B$ when $\theta > \theta^*(\psi_1, \psi_2)$, and $G^C = G^B = G(\psi_2)$ when $\theta < \theta^*(\psi_1, \psi_2)$.

Proof of Lemma 2. Recall that $\hat{b}_N = [\hat{\pi}_N^1 - \hat{\pi}_N^2]^+$, $b_N^* = [\pi_N^1(\psi_1, \theta) - \pi_N^2(\psi_2, \theta)]^+$, and $\hat{\pi}_N^2 = \pi_N^2(\psi_2, \theta)$. By Table 2, $\hat{\pi}_N^1$ is increasing in $\psi$ (by simply checking the first-order derivative) and thus $\hat{\pi}_N^1(\psi_2) > \hat{\pi}_N^1(\psi_1)$ for all $0 < \psi_1 < \psi_2 < 1$. From Corollary 2, $\hat{\pi}_N^1(\psi) > \hat{\pi}_N^2(\psi)$ for any
\[\psi \in (0, 1) \text{ when } \theta \in (\frac{5\sqrt{6}}{6} - 2, \frac{1}{2}). \text{ Therefore, } \hat{\pi}_N^1 (\psi_2) > \hat{\pi}_N^2 (\psi_1). \text{ As } \hat{\pi}_N^2 (\psi_1) = \pi_N^* (\psi_1, \theta) \text{ (due to the same information structure), } \hat{\pi}_N^1 (\psi_2) - \hat{\pi}_N^2 (\psi_1) > \pi_N^* (\psi_1, \theta) - \pi_N^* (\psi_2, \theta). \text{ When } \theta \in (\frac{5\sqrt{6}}{6} - 2, \frac{1}{2}), \text{ since } \hat{b}_N > 0 \text{ by Corollary 2, } IR^B = \hat{b}_N > b^*_N \geq \min\{b^*_M, b^*_N\} = IR^C \text{ for all } 0 < \psi_1 < \psi_2 < 1.

\textbf{Proof of Lemma 3.} We plug in } p^*_M (\psi, \theta) \text{ and } p^*_N (\psi, \theta) \text{ from Eq.}(5) \text{ into Eq.}(13), \text{ and want to show } \frac{\partial^2}{\partial \psi} CS (\psi, \theta) < 0. \text{ (i) When } \psi < \frac{1}{3}, \frac{\partial^2}{\partial \psi} CS (\psi, \theta) = \frac{(1-2\theta)F (\psi, \theta)}{2(1-\theta)[3(1-\theta)(1-\psi)+4\theta\psi]^3}, \text{ where } F (\psi, \theta) \text{ is a polynomial function of } \psi \text{ and } \theta \text{ with degree of } 7 (\theta^4 \psi^3). \text{ As we can show, for } \forall \psi \in (0, \frac{1}{3}), \frac{\partial^2}{\partial \psi} F (\psi, \theta) > 0 \text{ when } \theta < \frac{3}{7}, \text{ and } \frac{\partial^2}{\partial \psi^2} F (\psi, \theta) > 0 \text{ when } \theta > \frac{3}{7}. \text{ In other words, } F (\psi, \theta) \text{ is either increasing or convex in } \psi. \text{ Notice that } F (0, \theta) = (1-\theta)^2(-33-8\theta+22\theta^2) < 0 \text{ and } F (\frac{1}{3}, \theta) = \frac{9}{14}(3-\theta)^2(33-37\theta+10\theta^2) < 0 \text{ for } \forall \theta \in (0, \frac{1}{3}), \forall \psi \in (0, \frac{1}{3}). \text{ (ii) When } \psi > \frac{1}{3}, \text{ we have } CS (\psi, \theta) = \frac{(1-2\theta)(1+2\theta)}{8(1-\theta)} (1-\psi), \text{ which is decreasing in } \psi. \text{ Altogether, according to Propositions 1 and 2, } CS^C = CS (\psi_1, \theta) > CS (\psi_2, \theta) = CS^B \text{ when } \theta > \theta^* (\psi_1, \psi_2), \text{ and } CS^C = CS^B = CS (\psi_2, \theta) \text{ when } \theta < \theta^* (\psi_1, \psi_2).

\textbf{Proof of Proposition 5.} By Lemma 3, } CS^C > CS^B \text{ when } \theta > \theta^* (\psi_1, \psi_2). \text{ Since } LG = g (CS) \text{ is strictly increasing in } CS, \text{ } LG^C > LG^B \text{ when } \theta > \theta^* (\psi_1, \psi_2). \text{ Combining the result from Lemma 2, we have } IR^C < IR^B \text{ and } LG^C > LG^B \text{ when } \theta > \max\{\theta^* (\psi_1, \psi_2), \frac{5\sqrt{6}}{6} - 2\}.

\textbf{Proof of Proposition 6.} We first derive the second-stage equilibrium prices. When } M \text{ wins the first sponsored slot, for } \psi_2 < \frac{1}{3(1-\lambda)},

\[
\begin{align*}
\hat{p}_M^1 & = \frac{(1-\lambda)\psi \phi_2 + 2-\theta}{4(1-\lambda)\psi \theta + 3(1-\lambda)(1-\psi \phi_2)(1-\theta) + 3\lambda(1-\theta)} \\
\hat{p}_N^2 & = \frac{(1-\lambda)\psi \theta (3\theta - 1) + 1-\theta^2}{(1-\theta)[4(1-\lambda)\psi \theta + 3(1-\lambda)(1-\psi \phi_2)(1-\theta) + 3\lambda(1-\theta)]}.
\end{align*}
\]

When } N \text{ wins the first sponsored slot,

\[
\begin{align*}
\hat{p}_M^2 & = \frac{\lambda \psi (1-\theta) + 2-\theta}{(1-\theta)(\lambda \psi + 3)} \\
\hat{p}_N^1 & = \frac{\lambda \psi \phi_2 + 1+\theta}{(1-\theta)(\lambda \psi + 3)}.
\end{align*}
\]

(i) We can calculate } N \text{'s equilibrium profit in both cases: } \hat{\pi}_N^1 = \hat{p}_N^1 D_N^1 (\hat{p}_M^1, \hat{p}_N^2) \text{ and } \hat{\pi}_N^2 = \hat{p}_N^2 D_N^2 (\hat{p}_M^2, \hat{p}_M^1) \text{ according to Eq.}(18) \text{ and Eq.}(17). \text{ We then have } \hat{\Delta} \hat{\pi}_N (\psi_2, \theta, \lambda) = \hat{\pi}_N^1 - \hat{\pi}_N^2, \text{ and } \frac{\partial}{\partial \lambda} \hat{\Delta} \hat{\pi}_N (\psi_2, \theta, \lambda) \big|_{\lambda=0} = \frac{\psi_2 F (\psi_2, \theta)}{2(1-\theta)(4\psi_2 + 3(1-\psi_2)(1-\theta))}, \text{ where } F (\psi_2, \theta) \text{ is a polynomial function of } \psi_2 \text{ and } \theta \text{ with degree of } 8 (\theta^2 \psi_2^3). \text{ We can show that } \frac{\partial^2}{\partial \psi_2^2} F (\psi_2, \theta) = f (\theta) [4\psi_2 + 3(1-\psi_2)(1-\theta)], \text{ where } f (\theta) \text{ is some expression that only contains } \theta. \text{ For a given } \theta, \frac{\partial^2}{\partial \psi_2^2} F (\psi_2, \theta) \text{ does not change sign for } \forall \psi_2 \in (0, 1); \text{ that is, } \frac{\partial}{\partial \psi_2} F (\psi_2, \theta)

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is monotonic in $\psi_2$. We can check that $\frac{\partial}{\partial \psi_2} F (0, \theta) < 0$ and $\frac{\partial}{\partial \psi_2} F (1, \theta) < 0$, $\forall \theta \in (0, \frac{1}{2})$. Therefore, $\frac{\partial}{\partial \psi_2} F (\psi_2, \theta) < 0$ for $\forall \psi_2 \in (0, 1)$ and $\forall \theta \in (0, \frac{1}{2})$. Since we focus on the case where $\psi_2 < \frac{1}{3(1-\lambda)}$ ($= \frac{1}{3}$ at $\lambda = 0$) and we notice that $F (\frac{1}{3}, \theta) = \frac{2}{27} (3-\theta)^2 (32\theta^3 + 244\theta^2 - 523\theta + 129)$ crosses zero from above once when changing $\theta$ at $[0, 1]$. $F (\frac{1}{3}, \theta) > 0$ when $\theta \in (0, \theta_0)$. By monotonicity of $F$, $F (\psi_2, \theta) > 0$ and thus $\frac{\partial}{\partial \lambda} \Delta \pi_N (\psi_2, \theta, \lambda) \mid_{\lambda=0} > 0$, for $\forall \psi_2 \in (0, \frac{1}{3(1-\lambda)})$ and $\forall \theta \in (0, \theta_0)$. We can further check that $\theta_0 < \theta^* (0, \psi_2)$ (as defined by Eq.(7)) for the given parameter region, which validates the presumed bidding outcome (i.e., $b_N < b_M$).

(ii) Substituting equilibrium prices in Eq.(22) into the expression of equilibrium consumer surplus in Eq.(19), we can derive $\frac{\partial}{\partial \lambda} \tilde{C} S (\psi_2, \theta, \lambda) \mid_{\lambda=0} = \frac{(1-2\theta)\psi_2 G (\psi_2, \theta)}{2(1-\theta)[4\psi_2 \theta + 3(1-\psi_2)(1-\theta)]^3}$, where $G (\psi_2, \theta)$ is a polynomial function of $\psi_2$ and $\theta$ with degree of 7 ($\theta^4 \psi_2^3$). As we can show, for $\forall \psi_2 \in \left(0, \frac{1}{3(1-\lambda)}\right)$, $G (\psi_2, \theta)$ is decreasing in $\psi_2$ when $\theta < \frac{2}{7}$, and $G (\psi_2, \theta)$ is concave in $\psi_2$ when $\theta > \frac{2}{7}$. Notice that $G (0, \theta) = (1-\theta)^2 (33 + 8\theta - 22\theta^2) > 0$ and $G (\frac{1}{3}, \theta) = \frac{2}{27} (3-\theta)^2 (33 - 37\theta + 10\theta^2) > 0$, we can conclude that $G (\psi_2, \theta) > 0$ and hence $\frac{\partial}{\partial \lambda} \tilde{C} S (\psi_2, \theta, \lambda) \mid_{\lambda=0} > 0$ for $\forall \psi_2 \in \left(0, \frac{1}{3(1-\lambda)}\right)$ and $\forall \theta \in (0, \theta_0)$.

(iii) Substituting equilibrium prices in Eq.(22) into the welfare function in Eq.(20), we can derive $\frac{\partial}{\partial \lambda} \tilde{W} (\psi_2, \theta, \lambda) \mid_{\lambda=0} = \frac{\psi_2 H (\psi_2, \theta)}{2(1-\theta)[4\psi_2 \theta + 3(1-\psi_2)(1-\theta)]^3}$, where $H (\psi_2, \theta)$ is a polynomial function of $\psi_2$ and $\theta$ with degree of 8 ($\theta^5 \psi_2^3$). Following similar arguments as in (i), we can show that $H (\psi_2, \theta)$ is decreasing in $\psi_2$ for $\forall \psi_2 \in (0, 1)$ and $\forall \theta \in (0, \theta_0)$. Furthermore, $H (\frac{1}{3}, \theta) = \frac{2}{27} (3-\theta)^2 \left[\theta \left(12 \left(\theta - \frac{3}{7}\right)^2 - 8\right) + 3\right] > 0$ for $0 < \theta < \frac{1}{2}$. Therefore, $H (\psi_2, \theta) > 0$, and thus $\frac{\partial}{\partial \lambda} \tilde{W} (\psi_2, \theta, \lambda) \mid_{\lambda=0} > 0$ for $\forall \psi_2 \in \left(0, \frac{1}{3(1-\lambda)}\right)$ and $\forall \theta \in (0, \theta_0)$.

(iv) Substituting equilibrium prices in Eq.(22) into the expression of sales Gini coefficient in Eq.(21), we can derive $\frac{\partial}{\partial \lambda} \tilde{G} (\psi_2, \theta, \lambda) \mid_{\lambda=0} = \frac{\psi_2 \psi_2 (A \psi_2^2 + B \psi_2 + C)}{(\theta^2 \psi_2^2 + B \psi_2 - 5 \psi_2 + 3 \psi_2 - 3)^3}$, where $A = \theta^3 - 15 \theta^2 + 13 \theta - 3$, $B = -2 \theta^3 + 18 \theta^2 - 22 \theta + 6$, and $C = \theta^3 - 7 \theta^2 + 11 \theta - 5$. As we can verify that $B^2 - 4AC < 0$ and $A < 0$, $\frac{\partial}{\partial \lambda} \tilde{G} (\psi_2, \theta, \lambda) \mid_{\lambda=0} < 0$ for $\forall \psi_2 \in \left(0, \frac{1}{3(1-\lambda)}\right)$ and $\forall \theta \in (0, \theta_0)$.