When Should Software Firms Commercialize New Products via Freemium Business Models?

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Abstract

In the software industry, a challenge firms often face is how to effectively commercialize innovations. An emerging business model increasingly embraced by entrepreneurs, called *freemium*, combines "free" and "premium" consumption in association with a product or service. In a nutshell, this model involves giving away for free a certain level or type of consumption while making money on premium consumption. We develop a unifying multi-period microeconomic framework with network externalities embedded into consumer learning in order to capture the essence of conventional for-fee models, several key freemium business models such as feature-limited or time-limited, and uniform market seeding models. Under moderate informativeness of word-of-mouth signals, we fully characterize conditions under which firms prefer freemium models, depending on consumer priors on the value of individual software modules, perceptions of cross-module synergies, and overall value distribution across modules. Within our framework, we show that uniform seeding is always dominated by either freemium models or conventional for-fee models. We further discuss managerial and policy implications based on our analysis. Interestingly, we show that freemium, in one form or another, is always preferred from the social welfare perspective, and we provide guidance on when the firms need to be incentivized to align their interests with the society's. Finally, we discuss how relaxing some of the assumptions of our model regarding costs or informativeness and heterogeneity of word of mouth may reduce the profit gap between seeding and the other models, and potentially lead to seeding becoming the preferred approach for the firm.

Key words: software markets; freemium business models; economics of IS; software diffusion and adoption; software versioning; seeding strategies; product sampling

1 Introduction

Efficient diffusion of a product innovation represents a challenge faced by many firms in the software industry. Conventional models, whereby the entire product or each separate module comes with an associated price, represent a legacy of the early stages of the software industry that still endures to the present day (e.g., various versions of Apple or Microsoft Windows operating systems). Nevertheless, the software market infrastructure and ecosystem have experienced a sea change over the past couple of decades. On one hand, information technology performance-to-price ratio increased significantly and user interfaces became much more friendly, accelerating the adoption of computers towards supporting activities on both professional and personal levels, which, in turn, led to software becoming ubiquitous to the functioning of our society. According to Datamonitor (2009), the size of the rapidly growing global software market was $303.8 billion in 2008 and is estimated to reach $457 billion by 2013. On the other hand, Internet penetration and usage grew at a staggering rate, with current statistics reporting over 1.96 billion Internet users and over 681 million interconnected Internet hosts worldwide (Internet World Stats 2010, ISC 2010). This led to the emergence of online software distribution models (e.g., online software marketplaces such as Apple App Store, Google Apps Marketplace, Microsoft online store, or repositories such as SourceForge.net), online software consumption models (e.g., software-as-a-service offerings such as Salesforce.com CRM suite or GE's Centricity Electronic Medical Records service), as well as online feedback models that strengthen the word-of-mouth effects (Dellarocas 2003).

In parallel with this paradigm shift, a new software business model, called *freemium* (Anderson 2009), has emerged, combining "free" and "premium" consumption in association with a product or service. In a nutshell, the model involves giving away for free a certain level or type of consumption while making money on premium consumption. Freemium models are spreading quickly in the software industry, especially among Web start-ups (Miller 2009).

Two of the most commonly encountered freemium models are *feature-limited freemiums* (FLF) and *time-limited freemiums* (TLF) (Anderson 2009, p. 245). FLF models involve offering a basic version of the product with limited functionality for free, while charging for additional features in the premium version. Yahoo! for example, offers basic email service for free, but charges for its premium Mail Plus service that includes value-adding features such as live customer care and mail forwarding. Skype, an Internet-based telecommunications company, allows free PC-to-PC voice communication and free text messaging, but charges users for voice calls to landline or mobile phone numbers. Many PC video game developers use free demos allowing limited gameplay (sometimes referred to as “lite” versions of the game) in order to entice potential customers into buying the full version. This approach to selling games is very popular in online application stores for mobile devices (e.g., iTunes App store for iPhone and iPad users). TLF models, on the other hand, allow users free access to the full version of the software product, but only for a limited period of time. At the expiration of the free trial period, the software locks itself and users have to purchase a registration key in order to continue to use it. For example, Adobe Photoshop CS 5 and Microsoft Office 2010 come with a 30-day and a 60-day free trial, respectively.

Hybrids between FLF and TLF models also exist. For example, for the past several years, Adobe’s strategy was to offer its basic pdf reader for free. However, it charges for the premium features in its Acrobat full-version product but it allows users to download a 30-day free trial of this premium version with complete functionality. Personalized Internet radio service Pandora, which allows users to customize their stations, offers free basic service (access via web browser, 40 listening hour cap per month, lower sound quality) while charging for its premium Pandora One service (desktop application, unlimited listening, higher sound quality). TexPoint add-in for Microsoft Powerpoint, which facilitates embedding LaTeX into slides, locks some of the premium features after a free trial while basic functionality continues to be available for free.

Freemium models challenge not only conventional for-fee business models but also *market seeding models* which have also been employed in software commercialization. Market seeding is a business model that is different than conventional marketing approaches such as advertising and free demonstration, involving firms giving away the unlimited-time full-functionality product to a percentage of the addressable market in order to jump-start adoption. Autodesk, a leader in professional design and engineering software, announced in 2009 an initiative to seed 100 early-stage clean tech companies with free software bundles each worth approximately $150,000 (Autodesk 2009). Microsoft, through its DreamSpark student program, offers free professional-grade software developer and designer tools to millions of college and high school students around the world (Microsoft 2008).

The feasibility of the freemium models is predicated on many features of software products. First, software products nowadays are increasingly built using a modular architecture which, in turn, facilitates grouping, separating, or locking certain features. Second, it is relatively easy to embed in software programs trial-expiration clocks associated with feature-locking mechanisms. Third, software products are digital goods with negligible marginal reproduction cost, that can be provided in unlimited supply and can be “shipped” via relatively cheap online distribution channels. This way, the great majority of interested customers have access to the free consumption opportunities embedded in the freemium offering. Fourth, software products often belong to the category of experience goods. By trying (sampling) the product or part of it before committing to any purchase, consumers could learn more about the quality and other attributes (such as speed, functionality, and features) of the software, capabilities of related modules, compatibility issues, hardware requirements, etc.

It remains largely unknown how well these freemium models fare, compared to conventional and well established business models. As illustrated by the examples above, freemium models are applied nowadays in both software-as-a-service markets as well as markets for software products characterized by a one-time purchase and designed to run autonomously on the consumers' hardware. In this paper, we focus on the latter. Motivated by inquiries from entrepreneurs, we advance a unified multi-period framework that accounts for word-of-mouth effects and cross-module synergies and can capture the essence of several software business models, including freemium and seeding ones. Under moderate informativeness of word-of-mouth signals, we use this framework in particular to derive the equilibrium for each model and fully characterize conditions under which freemium models are superior to conventional charge-for-everything and uniform seeding models. We further compare freemium models against each other. To the best of our knowledge, this represents the first extensive analytical benchmarking study that juxtaposes for-fee, freemium, and seeding models. While most of the existing literature focuses on illustrating how individual variants of free consumption models may outperform conventional models, we take the analysis one step further by exploring the differences among several such free consumption models. Our analysis reveals several new and interesting results. In our setting, from the firm's perspective, we find that while seeding models are always dominated, the conventional for-fee model and the freemium models can each be optimal in separate regions of the feasible space. We also show that the relative performance difference between the freemium models can be quite large, thus highlighting on the firm side the managerial importance of understanding the market dynamics induced by each of these business models. Furthermore, we address policy implications by demonstrating that switching from conventional or seeding models to freemium models always increases social welfare. Matching welfare results with the profit analysis, we also explore cases when the firm should be subsidized (i.e., when the interests of the firm and the society are not well aligned). Last, we explore how relaxing some of the assumptions of our model regarding costs or informativeness and heterogeneity of word of mouth may reduce the performance gap between seeding and the other models and may potentially lead to seeding being preferred by the firm.

The rest of the paper is organized as follows. §2 presents a summary of the relevant literature. In §3, we introduce the general models. In §4, §5, and §6, we derive the optimal strategy of the software firm under baseline, freemium, and seeding models. In §7, we run comparative statics on these strategies. In §8, we conduct a social welfare analysis and explore policy implications. In §9, we explore how the performance of the seeding strategy is impacted by the relaxation of some of our model assumptions. In §10, we summarize the conclusions of this study and present directions for further research. For brevity, all proofs have been put in the Online Supplement.

2 Literature Review

In this study, we draw mainly on five streams of research in IS economics and marketing: (i) two-sided markets and platform strategies, (ii) open source software, (iii) software market segmentation via quality degradation and product versioning, (iv) product sampling and free demonstration, and (v) market seeding.

First, the underlying concept behind FLF models is linked to the burgeoning literature on two-sided markets and platform strategies, since the latter explores how firms may be able to boost adoption of premium products or features (the "money" side) by subsidizing the adoption of the basic functionality (the "loss" side). In doing so and compared with charging both sides, the firms may profit more overall due to cross-side positive network effects. This literature largely focuses on pricing strategies for distinct but complementary products/markets, with significant interests in identifying the "money" side (Rochet and Tirole 2003, 2006, Parker and Van Alstyne 2005). We extend this line of research in the context of software markets by inspecting how an FLF strategy that involves subsidizing the basic functionality fares compared to other free-consumption offerings such as TLF or seeding.

FLF models are encountered in both open source and proprietary software markets. In the open source software markets, the source code is offered for free and the originator (and sometimes contributing developers as well) may make profit on consulting, integration, support, or other complementary products or services (Iansiti and Richards 2006, Haruvy et al. 2008). At the individual level, less visible contributing developers may exhibit an FLF-like behavior, whereby, even if their contribution is not paid, they may still choose to volunteer it in order to obtain community exposure and skill recognition, which may translate into better job prospects, increased income, and/or new business opportunities

(Lerner and Tirole 2002, Roberts et al. 2006). The originator chooses whether or not to open the project at the beginning of or during the development stage, in order to allow for the creation and assimilation of outside code contributions. The choice of proprietary versus open source will thus influence the quality and versatility of the end product. We direct interested readers to von Krogh and von Hippel (2006) for a review and classification of the existing literature on open source software. In our paper, we focus on proprietary software, where, unlike in the open source case, the decision regarding the business model can be postponed until the end of the development stage and the quality, development cost, and functionality level cannot be altered by existing users.

The third stream of extant work relevant to our research is on proprietary software market segmentation via versioning and quality degradation, which in turn draws on the well known economics literature on second-degree price discrimination (e.g., Mussa and Rosen 1978). In FLF models, vertical differentiation is achieved by offering the lower "quality" version (where quality can be measured in terms of performance, functionality, or content limitations) at no charge. There exists a rich literature (e.g., Raghunathan 2000, Bhargava and Choudhary 2001, 2008, Wei and Nault 2005, Chen and Seshadri 2007, Jones and Mendelson 2010) that studies optimal software versioning under various utility structures and market assumptions. Ghose and Sundararajan (2005) complement this line of work by empirically estimating the extent of quality degradation associated with software versioning, using a quadratic utility structure. Wu and Chen (2008) further show how versioning can be used to deter digital piracy. Riggins (2003) and Cheng and Tang (2010) investigate cases where the low-quality product is offered for free in the context of unique web content and software products, respectively. In the former study, the firm monetizes giveaways through advertising revenues generated by the users of free websites, whereas in the latter (extending the models in Conner (1995) and also adding an aggregate consumer usage cost) the firm trades off consumer valuation upshift due to positive network effects vs. lost sales due to giveaways of the low-quality version. In Cheng and Tang (2010), a critical condition for the optimality of FLF strategy is the presence of strictly positive software usage costs at the individual consumer level. We show, however, that FLF can also be optimal under negligible software usage cost when there are network externalities embedded in valuation learning.

A common assumption of all the above reviewed versioning models is that customers have full information regarding the value of each product version, and they self-select into purchase groups according to the menus of price and quality offered by the vendors. Essentially, these models assume a pre-existing first period when customers were allowed to test fully functional versions prior to market release, followed by a second period when the versioned product is commercialized. While this stream of literature advances the theory behind optimal versioning, it does not explore in the first place whether or not the firm should fully inform customers with respect to product value prior to market release. Moreover, most of these models (with the exception of Raghunathan 2000, who presents a two-period model without network externalities) consider one-period frameworks, abstracting from adoption dynamics over time. In that sense, network effects are captured at the utility level (if at all) as usage benefits, ignoring community-induced consumer valuation learning over time via word of mouth. By employing a two-period framework with word-of-mouth effects to inspect how valuation learning and imperfect information on the consumer side affect the FLF offering and its performance relative to perfect information approaches such as TLF, we bring a significant contribution to the existing literature on software versioning.

The fourth stream of literature relevant to our work is related to product sampling and free demonstration. These well studied marketing strategies are particularly appealing to digital goods, many of which are *experience* goods whose value is learnt by customers via trying the good itself or a version of it (Chellappa and Shivendu 2005). In practice, freemium models do capitalize on this important characteristic of digital goods. While under TLF consumers gain full access to the software functionality through a free trial for a limited period of time and learn the true value of each feature, under FLF, they only adjust their priors on the value of premium features based on experiencing basic features but may not reach the true value. Therefore, in digital goods markets, net of advertising, word of mouth effects, or direct network effects, *firms can influence individual consumers' product value expectations by controlling the degree and type of free consumption*. Thus, when comparing strategies side by side, the firm must account for the impact of its chosen business model on consumer perceptions, since, in turn, these perceptions affect sales.

Acknowledging this effect, the literature on product sampling and free demonstration explores how firms can influence adoption by educating (some of) the customers on the value of the product. One line of work (e.g., Jain et al. 1995, Heiman and Muller 1996, Heiman et al. 2001) accounts for network effects and establishes that customers change their priors on the product value after sampling it. However, most of these models consider that sampling and free demonstrations are made available only to a limited audience due to substantial replication and distribution costs associated with physical goods. Bawa and Shoemaker (2004) consider the extreme case where samples are offered to the entire addressable market. In all of the above models, price is treated as exogenous rather than the firm's strategic choice. Chellappa and Shivendu (2005) and Cheng and Liu (2010) extend this line of work by exploring sampling under endogenous pricing. The former study models a vertically differentiated market for digital goods where consumers do not know the true product value but can sample the goods through pirated versions. The latter study presents an analysis of the optimality of free trial software that is related to ours but approaches the problem under a different setting. Cheng and Liu (2010) model the tradeoff between reduced uncertainty and demand cannibalization, extending Cheng and Tang (2010) by adding free trial duration, thus allowing consumer belief updating over time. Restricting their analysis to the case when consumers underestimate the value of the product, the authors identify conditions under which time-locked free trail is preferable and explore optimal stoping time for free trial. They also present a numerical, but not analytical, profit comparison between limited-time and limited-version (with perfect information) models. Again, similar to Cheng and Tang (2010), in the absence of software usage cost, limited-time approach will always dominate limited-version approach since the firm cannot make any profit on the latter. Our setting and analysis are rather different than theirs. Among others, we compare analytically a larger set of business models (including seeding strategies), and our modeling framework accounts for cross-module synergies that impact valuation learning and incorporates multi-period adoption dynamics based on word-of-mouth effects. In particular, when consumers underestimate the value of the product, we prove that FLF can dominate TLF (and all other models) even under negligible software usage costs.

The fifth stream of extant work relevant to our research is on market seeding, whereby a ratio of the potential customers receive the full product for free. In that sense, seeding is another marketing strategy for the firm to influence consumer priors in order to jump-start adoption. Jiang and Sarkar (2003) explore the effect of limited product giveaways on future adoption and net present value of future sales under an exogenous pricing rule. Lehmann and Esteban-Bravo (2006) inspect optimal seeding under endogenized dynamic pricing, variable costs, and network externalities. Galeotti and Goyal (2009) analyze optimal seeding in a social network with a graph-like topology where decisions of individual customers are influenced by their immediate contacts. None of these studies, however, addresses how seeding models fare compared to freemium and conventional for-fee models, which we do.

In summary, we contribute to the previous literature by integrating several of the above reviewed isolated streams of research. We develop a general multi-period adoption framework with bi-dimensional consumer valuation learning accounting for both firm-induced effects (achieved by controlling the degree and type of free consumption) and community-induced word-of-mouth effects, thus capturing how consumer behavior evolves over time under different business models. Based on this unified framework, we formulate, solve, and compare five business models: conventional for-fee, feature-limited freemium, time-limited freemium, simple uniform seeding without first period selling, and complex uniform seeding allowing first period selling. In addition to the above mentioned contributions to each of the related streams of research, to the best of our knowledge, this is the first extensive analytical benchmarking of these business models, thus advancing our understanding of the firm and society benefits resulting from offering one form or another of *free consumption*. Previous literature mostly compares individual free consumption models against the conventional for-fee model, but these studies do not compare and contrast analytically the profit and social welfare performance of free consumption models among themselves, in part because of the lack of an integrating framework.

3 Models

We assume that a firm has developed a software product that has two modules A and B, and is exploring the most effective way to commercialize it. Basic functionality is coded in module A, while module B incorporates premium features or content. The product (both modules) has a life span of two periods (after which it becomes obsolete or irrelevant) and the firm commits to selling the entire product (both A and B) or only part of it (give away A for free and sell only B) at a fixed one-time price p. Customers who purchase the product in period 1 use it also in the second period at no additional charge. Examples from software business practice where constant price over time has been combined with freemium strategies include AutoCAD design software, DivX video software, WinEdt LaTeX scientific text editor, etc.[1] The firm is considering among five potential business models:

(a) **Charge-for-everything (CE).** The firm sells both modules *together* (as one indivisible product) in both periods. No consumption degree or type is offered for free.

(b) **Feature-Limited Freemium (FLF).** The firm gives away module A and sells *only* module B in both periods.[2]

[1] For the past several years AutoCAD, DivX, and WinEdt have exhibited constant list prices. Autodesk charges $3995 for a perpetual AutoCAD license while also allowing for a 30-day free trial. DivX video software bundle, comprising of video player, web browser player, codecs and plugins for various video formats has a zero price tag for its basic version and a $19.99 price tag for its premium DivX Pro version. WinEdt is priced at $40 per individual, non-business license but comes with a 30-day free trial that consumers could use indefinitely, at the cost of enduring increasingly frequent pop-up reminders (after expiration of the free trial) to buy a license and register the product.

[2] It is worth mentioning that, in practice, in the case of FLF models, sometimes premium functionality is bundled together with basic functionality in the premium version of the product creating a complete stand-alone solution (e.g., Adobe Reader vs Adobe Acrobat). However, if A is offered for free, whether the premium functionality is delivered as an add-on (only B) or as a separate stand-alone solution (both A and B integrated in one product), consumer choice is the same due to incentive compatibility constraints.

(c) **Time-Limited Freemium (TLF).** The firm allows consumers to try both A and B for free but *only* during period 1, and sells both modules *together* (as one indivisible product) *only* during period 2.

(d) **Simple Seeding (SS).** The firm gives away for free the product to a percentage of the addressable market (uniformly across consumer types) in both periods, and then sells both modules *together* (as one indivisible product) to the remaining consumers *only* during period 2 .

(e) **Complex Seeding (CS).** The firm gives away for free the product to a percentage of the addressable market (uniformly across consumer types) in both periods, and in parallel, sells both modules *together* (as one indivisible product) during both periods to the remaining consumers.

The firm is a profit maximizer. All software development costs are sunk and it is assumed that the costs associated with opening and locking features are negligible.

We assume a normalized mass $m = 1$ of consumers with types θ distributed uniformly in the interval $[0, 1]$. A consumer of type θ derives *per-period* benefits $a\theta$ and $b\theta$ from using modules A and B, with $a, b > 0$. In that sense, type θ captures the marginal consumer willingness to pay for quality and functionality per unit of time. The resulting linear utility model is similar to the one in Chen and Seshadri (2007), with the only difference that earlier adopters get to use the product more compared to late adopters. We consider a general setting where parameters a and b capture an aggregate benefit from the modules and are not necessarily linear in the number of features included in each module, as users might value various features differently. Moreover, we include in parameter b all additional *cross-module* benefits that arise from using functionality in A and B jointly, which otherwise are not available to customers using module A as a stand-alone (e.g., under FLF model). For simplicity, we assume no time discounting.

We consider a market structure with information asymmetry. On one hand, we assume that prior to product introduction (before the beginning of period 1) potential customers do not know the true value for any of the modules but they have priors $a_0 = \alpha_a \cdot a$ and $b_0 = \alpha_b \cdot b$ for these values, with $\alpha_a,\ \alpha_b > 0$. We consider a general setting, allowing for $\alpha_a \neq \alpha_b$. If no module is offered for free (model CE), consumers maintain their prior throughout period 1, i.e. $a_1 = a_0$, $b_1 = b_0$. We assume the same is true for the unseeded portion of the market under seeding models (SS, CS). In particular, under CS, we assume that sales and seeding occur in parallel during period 1, and the word-of-mouth effect of seeding on consumer valuation learning manifests at the end of period 1, as further discussed in §6.2. Next, we assume that if A is offered for free (model FLF), then potential customers learn immediately value a and they update their estimate of b as $b_1 = (\alpha_b + \delta_b) \cdot b$, with $\alpha_b + \delta_b > 0$. With value of module A fully revealed, consumers may learn more about the value of premium features due to the existence of cross-module synergies. However, such an initiative on the firm's behalf may also push consumers further away from the true value b (e.g., when $\alpha_b \leq 1$ and $\delta_b < 0$, or when $\alpha_b \geq 1$ and $\delta_b > 0$). If both modules are offered for free in period 1 (model TLF), then consumers immediately update their priors to the true values, i.e., $a_1 = a$ and $b_1 = b$. On the other hand, we assume that the firm has full information about parameters a, b, α_a, α_b, and δ_b. Consistent with the literature (e.g., Chen and Seshadri 2007), we assume that the firm has incomplete information about consumers in the sense that it knows the consumer type distribution but does not know the specific type of any individual consumer.

Regardless of the chosen business model, at the beginning of period 2, consumers who have not purchased the software in period 1 further adjust their beliefs about the non-free modules based on *word-of-mouth effects* (network externalities at the valuation learning level) generated by existing adopters. Word-of-mouth effects are commonly considered to affect adoption by influencing consumers' perceptions of product value and attributes (Mahajan et al. 1984, Ellison and Fudenberg 1995) and are increasingly disseminated over the Internet (Dellarocas 2003, Duan et al. 2008, Trusov et al. 2009). For a given module $X \in \{A, B\}$ and a time period $t \in \{1, 2\}$, let $N_{t,X}$ denote the number of consumers who own module X by the end of period t (whether they purchased it, were seeded, or tried it in period t as part of a freemium offer). As a reminder, customers cannot own module B alone. Under CE, TLF, SS, and CS models we have $N_{t,A} = N_{t,B} = N_t$. Under FLF model, we have $N_{t,A} = 1 \geq N_{t,B}$. For a module $X \in \{A, B\}$ with true value $x \in \{a, b\}$ and prior $x_1 \in \{a_1, b_1\}$, consumers update their valuation perception at the beginning of period 2 as follows:

$$x_2 = x_1 \left(1 - N_{1,X}^{\frac{1}{w}}\right) + xN_{1,X}^{\frac{1}{w}} = x_1 + N_{1,X}^{\frac{1}{w}}(x - x_1), \tag{1}$$

where $\frac{1}{w}$ with $w > 0$ describes the degree of *informativeness* of the signal (in a different context, Wu et al. 2010 use a similar model where network size is replaced by project size or scope). Assuming a positive correlation between installed base and the number of reviews, our model captures the idea that sales, and, implicitly, consumer valuation learning and behavior, are affected by both review content and number of reviews (Chevalier and Mayzlin 2006, Dellarocas et al. 2007, Chen and Xie 2008). The larger the installed base of the product is, the more accurate the estimates of the potential customers with respect to a and b become. If adoption is very low in period 1, potential customers will not change their

	Before release	Beginning of period 1	Beginning of period 2
CE, SS, CS	$a_0 = \alpha_a \cdot a$ $b_0 = \alpha_b \cdot b$	$a_1 = \alpha_a \cdot a$ $b_1 = \alpha_b \cdot b$	$a_2 = a_1 + N_1^{\frac{1}{w}} \cdot (a - a_1)$ $b_2 = b_1 + N_1^{\frac{1}{w}} \cdot (b - b_1)$ $N_1 = N_{1,A} = N_{1,B}$
FLF	$a_0 = \alpha_a \cdot a$ $b_0 = \alpha_b \cdot b$	$a_1 = a$ $b_1 = (\alpha_b + \delta_b) \cdot b$	$a_2 = a$ $b_2 = b_1 + N_{1,B}^{\frac{1}{w}} \cdot (b - b_1)$
TLF	$a_0 = \alpha_a \cdot a$ $b_0 = \alpha_b \cdot b$	$a_1 = a$ $b_1 = b$	$a_2 = a$ $b_2 = b$

Table 1: Dynamics of customer perceptions under models CE, SS, CS, FLF, and TLF. In the paper, we focus on $w = 1$ case.

perception of the true module values much since they do not get a strong-enough signal. On the other hand, a high adoption in period 1 helps generating a lot of buzz about the product (module) and induces potential customers to adjust their priors very close to the true value. Note that equation (1) can also be extended to the free modules since word of mouth does not change the valuation if consumers learned the true value of either module A or B at the beginning of period 1 (i.e., $a_1 = a$ and/or $b_1 = b$) due to free consumption offers (e.g., via FLF or TLF). The evolution of the consumer perceptions over time for all models is summarized in Table 1.

We assume that customers derive the aggregate valuation adjustment quantity in response to market signals without having the ability to reverse engineer the word-of-mouth signals and retrieve the precise valuation parameters. In general, the degree of informativeness of peer reviews is not known to customers. Moreover, while offering a direction for the valuation adjustment, reviews in general contain a certain level of noise, thus obscuring the true value of the product. Furthermore, equation (1) generalizes to non-unit markets by replacing N_1 with $\frac{N_1}{m}$. While actual sales data might be available, market size may be unknown to consumers while firms may have a much better understanding of it.

Word of mouth can be more or less informative (Chen and Xie 2008). Given that $N_1 \leq 1$, large values of w ($w > 1$) are indicative of more informative signals, whereby consumers learn a lot from the first few reviews and derive diminishing marginal information from additional signals (absolute change in valuation is concave in N_1). Alternatively, low values of w ($w < 1$) describe less informative signals characterized by increasing marginal information derived from additional reviews. In the main part of the paper, for tractability, we focus on the case of an intermediate degree of informativeness $w = 1$ of the word of mouth, which allows us to capture relevant tradeoffs between learning via sampling and learning via reviews. Similar linear parametrizations of valuation learning (but with respect to sample size and trial time) have also been used in Chellappa and Shivendu (2005) and Cheng and Liu (2010). We discuss uninformative and very informative signals in §9.1.

We point out that our models can account for both *underestimation* ($\alpha_a < 1$, $\alpha_b < 1$, or $\alpha_b + \delta_b < 1$) and *overestimation* ($\alpha_a > 1$, $\alpha_b > 1$, or $\alpha_b + \delta_b > 1$) of the product value. Regardless of whether consumers overestimate or underestimate the product, the word-of-mouth effects push their perceptions of the true module value in the right direction.[3] Underestimation of module value usually occurs when consumers do not know all features embedded in the module and do not realize its full potential. Overestimation can occur when consumers initially expect to use the software more than they would do in reality. Very large values of α_a or α_b may describe situations when the product contains flaws (bugs or compatibility issues) that prevent it from delivering its intended output. In that case, true values a and b are very small compared to consumer priors. However, after one period, customers who ended up buying the faulty software report existing issues and other consumers learn about the problems and adjust their priors downwards. Overestimation may also occur when the product does not fit consumer tastes or preferences according to their expectations (Chellappa and Shivendu 2005).

4 Baseline Scenario - *CE* Model Equilibrium

We first derive the equilibrium for the conventional CE business model. Define $\gamma = \frac{b}{a} > 0$ as ratio of the true values (to consumers) of the two modules, and c and α as follows:

$$c = a + b \qquad \text{and} \qquad \alpha = \alpha_a \cdot \frac{a}{c} + \alpha_b \cdot \frac{b}{c} = \alpha_a \cdot \frac{1}{1+\gamma} + \alpha_b \cdot \frac{\gamma}{1+\gamma}. \tag{2}$$

Parameter c captures the value of the full product, including premium features, and consumers have an initial prior $c_0 = \alpha c$ on this value. For consistency with notation in §3, we define $c_1 = a_1 + b_1$, and $c_2 = a_2 + b_2$. It immediately follows that:

$$c_1 = c_0 = \alpha c \qquad \text{and} \qquad c_2 = c_1 + (c - c_1) \cdot N_1. \tag{3}$$

Under one-time fixed pricing, consumers will never consider at the beginning of period

[3] Note that this is different than updating the valuation of B in response to experiencing A for free (model FLF - beginning of period 1). In that case, consumers still have not experienced B but functionality in module A gives them additional signals about the potential benefits coming with functionality in module B. On the other hand, word of mouth is spread by *fully informed* customers who were seeded with or bought the premium product in period 1 and have experienced features in both modules A and B.

1 a strategy whereby to delay adoption until period 2 since $c_1 = \alpha c > 0$ and benefits accumulate over usage time. Therefore, at the beginning of period 1, potential adopters consider solely between adopting at the beginning of period 1 and not adopting at all.

The firm only considers the following feasible pricing strategies:

$$0 < p < 2c_1. \tag{4}$$

Otherwise, the firm is not making any positive profit. Let θ_1 be the *marginal* (lowest) type consumer that adopts the software in period 1. Then:

$$\theta_1 = \frac{p}{2c_1} = \frac{p}{2\alpha c}. \tag{5}$$

All consumers with type $\theta \geq \theta_1$ buy the software in the first period. Thus, $N_1 = 1 - \theta_1$. At the end of period 1, potential customers who have not adopted yet (i.e., those consumers with type $\theta < \theta_1$) update their estimate of c (more precisely, a and b) to c_2 (a_2 and b_2).

A consumer with type $\theta < \theta_1$ would adopt the software in period 2 and use it only for a single period of time if and only if $c_2\theta \geq p$. Let θ_2 be the *marginal* existing adopter at the end of period 2, regardless of when she adopted the product. Then:

$$\theta_2 = \begin{cases} \theta_1 & , \text{ if } \frac{p}{c_2} \geq \theta_1, \\ \frac{p}{c_2} & , \text{ otherwise.} \end{cases} \tag{6}$$

The installed base at the end of period 2 is $N_2 = 1 - \theta_2$. The following lemma captures the conditions for adoption in period 2.

Lemma 1. *Under CE model, for any feasible price* $p \in (0, 2c_1)$*, adoption extends to the second period* $(0 \leq \theta_2 < \theta_1)$ *if and only if* $c_2 > 2c_1$*.*

Lemma 1 illustrates the trade-off that potential customers are facing at the beginning of the second period if they did not adopt in the first period. On one hand, for the same price p, they can only benefit one period from using the product. On the other hand, their priors on c got adjusted from c_1 to c_2, and, as a result, their willingness to pay for one period of software use has changed. One interesting aspect captured in Lemma 1 is that consumers must at least double their priors on c as a result of network externalities in order to be willing to adopt in period 2. Weak adoption in period 1 does not provide potential customers with enough market feedback for them to consider a change in their adoption decision. Moreover, if consumers overestimated c in the beginning ($\alpha > 1$), then the updating at the end of period 1 lowers their priors ($c_2 < c_1$) due to calibration in the correct direction induced by word of mouth. In such a case, there is no adoption in period

2 since consumers' willingness to pay decreased. Replacing c_2 in Lemma 1 leads to the following corollary:

Lemma 2. *Under CE model, adoption extends to the second period if and only if consumers initially significantly underestimate the value of the software* $\left(\alpha < \frac{1}{2}\right)$ *and the price is sufficiently low* $\left(p < \frac{2c\alpha(1-2\alpha)}{1-\alpha}\right)$.

Lemma 1 explores period 2 adoption conditions in terms of updated estimate c_2, which in turn depends on period 1 adoption and, implicitly, on price. Lemma 2 advances our understanding of the adoption dynamics by illustrating that adoption never occurs in period 2 when consumer priors are high, price is high, or both.

We have fully characterized above the consumers' equilibrium response to any given price. Next, we solve the firm's profit maximization problem:

$$\max_{0<p<2c\alpha} p \cdot N_2. \tag{7}$$

This is a non-trivial problem since different pairs $\{\alpha, p\}$ induce different adoption patterns, as illustrated in Lemma 2. N_2 is a function of p that depends on whether there is adoption in the second period or not. We describe in the next proposition the equilibrium optimal pricing strategy for the firm.

Proposition 1. *Under CE model, the firm's optimal pricing strategy, and the profit and social welfare associated with it are:*

	$0 < \alpha < 13 - 4\sqrt{10}$	$13 - 4\sqrt{10} \leq \alpha$
p^*_{CE}	$\frac{2c\alpha}{1-\alpha}\left(1 - \sqrt{\frac{2\alpha}{1+\alpha}}\right)$	$c\alpha$
π^*_{CE}	$\frac{2c\alpha}{\left(\sqrt{1+\alpha}+\sqrt{2\alpha}\right)^2}$	$\frac{c\alpha}{2}$
SW^*_{CE}	$c\left[1 - \frac{1+2\alpha+2\alpha^2}{2(1+\alpha)\left(\sqrt{1+\alpha}+\sqrt{2\alpha}\right)^2}\right]$	$\frac{3c}{4}$
Adoption	*in both periods*	*only in period 1*

Figure 1 illustrates the firm's pricing strategy, profit, and induced adoption patterns.[4] In equilibrium, if consumer priors are low (i.e., $\alpha < 13 - 4\sqrt{10} \approx 0.351$), then the firm will

[4]When $\alpha = 13 - 4\sqrt{10}$, the firm can obtain the same profit via two different pricing strategies. At a high price, it can restrict adoption to period 1, whereas at a lower price it will induce adoption in both periods (with $N_2 - N_1 > 0$). All else being equal, we assume that the firm prefers to get the revenue earlier and keep the price high (perhaps as a statement of the quality of the product). Similar considerations are made in Propositions 2 and 5.

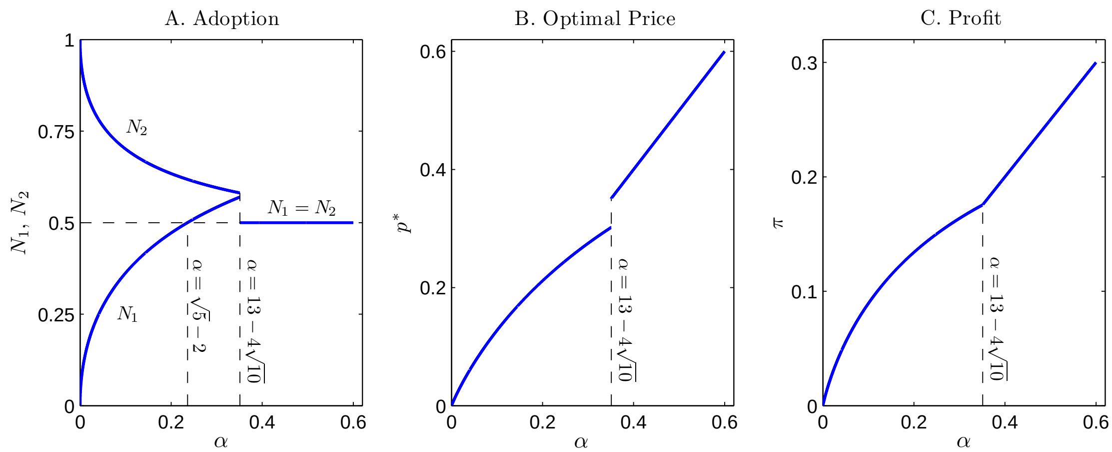

Figure 1: CE Equilibrium. Panels B and C use value $c = 1$ for illustration purposes.

price the product such that adoption occurs in two periods. In this case, even though later adoption involves shorter usage time, some of the customers make the purchase in period 2 after updating their priors at the end of period 1 due to word-of-mouth effects. When α is very small, the firm will employ a price that is low enough to generate a very small but significant enough mass of adopters in period 1 ($lim_{\alpha \downarrow 0} N_1 = 0$) who, in turn, fuel network externalities that make most potential customers adopt in period 2 ($lim_{\alpha \downarrow 0} N_2 = 1$). When consumer willingness to pay is higher, the firm will charge more (p^* is increasing in α) and there will be more period 1 adopters (N_1 is increasing in α) but fewer period 2 adopters ($N_2 - N_1$ is decreasing in α) and fewer overall adopters (N_2 is decreasing in α). Thus, as α grows, the firm exploits more the initial higher willingness to pay of consumers and less period 2 adoption as a result of word-of-mouth effects on valuation learning. In this region, N_2 is always greater than $\frac{1}{2}$. On the other hand, N_1 is lower than $\frac{1}{2}$ when $\alpha < \sqrt{5} - 2$, and higher than $\frac{1}{2}$ when $\alpha \in [\sqrt{5} - 2, 13 - 4\sqrt{10}]$. If $\alpha \geq 13 - 4\sqrt{10}$, the firm boosts the price linearly with α and exploits high willingness to pay solely in period 1 such that $N_1 = N_2 = \frac{1}{2}$. In particular, if consumers overestimate the true value of the software ($\alpha > 1$), the firm exploits this perception bias immediately through a higher price since it understands that period 1 consumption is critical in capturing the highest profit given that in period 2 word of mouth about lower-than-expected software quality will spread and will reduce per-period consumer willingness to pay.

5 Equilibria for Freemium Models

In this section we analyze the FLF and TLF models. From (2) we have $a = \frac{c}{1+\gamma}$ and $b = \frac{\gamma c}{1+\gamma}$. The derivation of the equilibrium solution for the FLF model is similar to that for the CE model by substituting $c \to b$ and $\alpha \to \alpha_b + \delta_b$. We also have to adjust the social welfare calculation by accounting for the fact that all consumers get module A for free for both periods.

Proposition 2. *Under FLF model, the firm's optimal pricing strategy, and the profit and social welfare associated with it are:*

	$0 < \alpha_b + \delta_b < 13 - 4\sqrt{10}$	$13 - 4\sqrt{10} \le \alpha_b + \delta_b$
p^*_{FLF}	$\frac{2(\alpha_b+\delta_b)\gamma c}{(1+\gamma)(1-\alpha_b-\delta_b)}\left(1-\sqrt{\frac{2(\alpha_b+\delta_b)}{1+\alpha_b+\delta_b}}\right)$	$\frac{\gamma c(\alpha_b+\delta_b)}{1+\gamma}$
π^*_{FLF}	$\frac{2(\alpha_b+\delta_b)\gamma c}{(1+\gamma)\left(\sqrt{1+\alpha_b+\delta_b}+\sqrt{2(\alpha_b+\delta_b)}\right)^2}$	$\frac{\gamma c(\alpha_b+\delta_b)}{2(1+\gamma)}$
SW^*_{FLF}	$\frac{c}{1+\gamma}\left[1+\gamma\left(1-\frac{1+2(\alpha_b+\delta_b)+2(\alpha_b+\delta_b)^2}{2(1+\alpha_b+\delta_b)\left(\sqrt{1+\alpha_b+\delta_b}+\sqrt{2(\alpha_b+\delta_b)}\right)^2}\right)\right]$	$\frac{c(4+3\gamma)}{4(1+\gamma)}$
Adoption	*in both periods*	*only in period 1*

The adoption pattern is similar to the one depicted in Figure 1, but with the horizontal axis describing $\alpha_b + \delta_b$ instead of α. Also, in this case, N_1 and N_2 should be replaced $N_{1,B}$ and $N_{2,B}$. When $\alpha_b + \delta_b \ge 13 - 4\sqrt{10}$, the firm's optimal pricing under FLF results in exactly half of the market being covered.

Under model TLF, adoption is restricted in period 2. Since consumers gain perfect information in period 1, their initial priors no longer influence adoption or, for that matter, firm's profit.

Proposition 3. *Under TLF model, the firm's optimal pricing strategy, and the profit and social welfare associated with it are* $p^*_{TLF} = \frac{c}{2}$, $\pi^*_{TLF} = \frac{c}{4}$, $SW^*_{TLF} = \frac{7c}{8}$.

Note that, under TLF model, we always have $N_2 = \frac{1}{2}$.

6 Equilibria for Seeding Models

In our framework, consumer learning depends on the existing network size but not on the types of customers who adopted before. If the firm had visibility into individual consumer types, then it would seed consumers who otherwise would not buy the product, instead of giving away the software to any high-type consumers. However, in a more realistic setting, the firm has limited knowledge of consumer types. For that reason, we consider *uniform seeding*, whereby $k\%$ of the addressable market receives the software for free in period 1 (with $k \in [0, 1)$) and each type is uniformly represented in the product giveaway pool.

6.1 Simple Seeding - *SS*

Under SS model, $N_1 = k$ since in period 1 we have only seeding but no selling. We again assume consumer valuation prior $c_1 = c\alpha$. The consumers who received the product for free start generating word of mouth about the overall quality of the product, leading the remaining $1-k$ consumers to update their willingness to pay to:

$$c_2 = c\alpha + kc(1-\alpha) = c[\alpha + k(1-\alpha)].$$

Then, in period 2, the remaining $1-k$ consumers have types uniformly distributed in the interval $[0,1]$. In order for any adoption to occur in the second period, by a similar argument as in §4, the firm will only consider feasible pricing strategies:

$$0 < p < c_2. \tag{8}$$

The marginal consumer, who is indifferent between adopting and not adopting, has type:

$$\theta_2 = \frac{p}{c_2} = \frac{p}{c[\alpha + k(1-\alpha)]}. \tag{9}$$

Then, we have $N_2 = (1-k)(1-\theta_2) + k$, accounting for the seeded consumers as well. However, in the profit function, we drop the seeded consumers since they do not generate any revenue for the firm, and obtain the expression:

$$\pi_{SS}(p,k) = p\,(N_2 - k) = p(1-k)\left(1 - \frac{p}{c[\alpha + k(1-\alpha)]}\right). \tag{10}$$

Proposition 4. *Under SS model, the firm's optimal seeding and pricing strategy, as well as the profit and social welfare associated with it are:*

	$0 < \alpha < \frac{1}{2}$	$\frac{1}{2} \leq \alpha$
k^*_{SS}	$\frac{1-2\alpha}{2(1-\alpha)}$	0
p^*_{SS}	$\frac{c}{4}$	$\frac{c\alpha}{2}$
π^*_{SS}	$\frac{c}{16(1-\alpha)}$	$\frac{c\alpha}{4}$
SW^*_{SS}	$\frac{c(11-16\alpha)}{16(1-\alpha)}$	$\frac{3c}{8}$

Interestingly, the firm only uses seeding when consumers significantly underestimate the true product value ($\alpha < \frac{1}{2}$). In such cases, the firm has an incentive to increase the consumer willingness to pay by generating word of mouth. Furthermore, when $\alpha < \frac{1}{2}$, this benefit exceeds the cost of uniform seeding, whereby the firm gives away the product also to some of the high-type customers who would have purchased it even under low priors. As expected, the optimal seeding ratio is decreasing in α, which illustrates the fact that the higher the consumers' priors on product value (and, implicitly, the higher the willingness to pay), the lower the need to seed the market.

6.2 Complex Seeding - *CS*

In this model, in period 1, in addition to seeding, the firm also sells the software. Word of mouth from the seeded community gets around even when nobody else purchases the product in the first period. Therefore, the feasible price condition (4) becomes:

$$0 < p < \max\{2c\alpha, c[\alpha + k(1-\alpha)]\}. \tag{11}$$

When constraint (11) is satisfied, adoption occurs in either period 1, period 2, or both. In particular, adoption (in addition to seeded customers) starts in period 1 if and only if $p < 2c\alpha$. We note that $2c\alpha < c[\alpha + k(1-\alpha)]$ if and only if $\alpha < \frac{k}{1+k}$. Therefore, for prices in the feasible range, we have:

$$\theta_1 = \begin{cases} \frac{p}{2c\alpha} & \text{, if } \frac{k}{1+k} \leq \alpha, \\ \frac{p}{2c\alpha} & \text{, if } 0 < \alpha < \frac{k}{1+k} \text{ and } p < 2c\alpha, \\ 1 & \text{, if } 0 < \alpha < \frac{k}{1+k} \text{ and } 2c\alpha \leq p < c[\alpha + (1-\alpha)k], \end{cases} \tag{12}$$

where $\theta_1 = 1$ indicates that negligible or no adoption occurs in period 1. Then, at the end of period 1, we have $N_1 = k + (1-k)(1-\theta_1) = 1 - (1-k)\theta_1$. It immediately follows that $c_2 = c\alpha + N_1 c(1-\alpha) = c - c(1-k)(1-\alpha)\theta_1$. The result in Lemma 1 still holds, i.e. adoption occurs in period 2 (i.e., $\theta_2 < \theta_1$) iff $c_2 > 2c_1$. Lemma 2 becomes:

Lemma 3. *Under CS model, for any feasible price satisfying* (11)*, adoption extends to the second period if and only if one of the following two cases occurs:*
(a) $\frac{k}{1+k} \leq \alpha < \frac{1}{2}$ *and* $p < \frac{2c\alpha(1-2\alpha)}{(1-\alpha)(1-k)}$, *or* *(b)* $0 < \alpha < \frac{k}{1+k}$.

For any fixed seeding ratio $k \in [0, 1)$, we derive the equilibrium solution below:

Lemma 4. *Under CS model, for a given seeding ratio $k \in [0, 1)$, the firm's optimal pricing strategy and the associated profit are:*

	$0 < \alpha < \tilde{\alpha}(k)$	$\tilde{\alpha}(k) \leq \alpha < \frac{13+2k+k^2-4\sqrt{10+2k^2+4k}}{(1-k)^2}$	$\frac{13+2k+k^2-4\sqrt{10+2k^2+4k}}{(1-k)^2} \leq \alpha$
$p^*_{CS}(k)$	$\frac{c[\alpha+(1-\alpha)k]}{2}$	$\frac{2c\alpha}{(1-\alpha)(1-k)}\left(1 - \sqrt{\frac{2\alpha}{2\alpha+(1-\alpha)(1-k)}}\right)$	$c\alpha$
$\pi^*_{CS}(k)$	$\frac{(1-k)c[\alpha+(1-\alpha)k]}{4}$	$\frac{2c\alpha\left(\sqrt{2\alpha+(1-k)(1-\alpha)}-\sqrt{2\alpha}\right)^2}{(1-k)(1-\alpha)^2}$	$\frac{c\alpha(1-k)}{2}$
Adoption	*only in per. 2*	*in both per. 1 and 2*	*only in per. 1*

where $\tilde{\alpha}(k)$ *represents the unique solution over the interval* $\left[\frac{-1-k^2+\sqrt{1+3k^2}}{1-k^2}, \frac{k}{3+k}\right]$ *to the equation* $h_k(\alpha) = 0$*, with* $h_k(\alpha) = [\alpha + (1-\alpha)k]\left(\sqrt{2\alpha + (1-\alpha)(1-k)} + \sqrt{2\alpha}\right)^2 - 8\alpha.$

This is a hard problem with a non-trivial proof. Existence and uniqueness of $\tilde{\alpha}(k)$ are proved in the Online Supplement in auxiliary Lemma A1. When $k = 0$, CS defaults to CE model, $\tilde{\alpha}(k) = 0$ (region $0 < \alpha < \tilde{\alpha}(k)$ does not exist), and we obtain the solution described in Proposition 1.

Solving for the optimal seeding ratio k^*_{CS}, we obtain the following surprising and elegant equilibrium solution:

Proposition 5. *Under CS model, the firm's optimal seeding and pricing strategy, as well as the profit and social welfare associated with it are:*

	$0 < \alpha < \underline{\alpha}$	$\underline{\alpha} \leq \alpha < 13 - 4\sqrt{10}$	$13 - 4\sqrt{10} \leq \alpha$
k^*_{CS}	k^*_{SS}	0	0
p^*_{CS}	p^*_{SS}	p^*_{CE}	p^*_{CE}
π^*_{CS}	π^*_{SS}	π^*_{CE}	π^*_{CE}
SW^*_{CS}	SW^*_{SS}	SW^*_{CE}	SW^*_{CE}
Adoption	*only in period 2*	*in both periods*	*only in period 1*

where $\underline{\alpha} \approx 0.065$ *is the unique solution to the equation* $g(\alpha) = 0$ *over the interval* $(0, 13 - 4\sqrt{10})$*, with* $g(\alpha) = \frac{1}{16(1-\alpha)} - \frac{2\alpha}{\left(\sqrt{1+\alpha}+\sqrt{2\alpha}\right)^2}$.

Existence and uniqueness of $\underline{\alpha}$ is proved in the Online Supplement in auxiliary Lemma A2. We note that, unlike in the case of model SS, under CS seeding is optimal only for very low priors, i.e., when consumers *severely* underestimate the value of the full product. For $\alpha \in (0, \underline{\alpha})$, the optimal seeding ratio $k^*_{SS} \in (0.46, 0.5)$, implying that, under CS, *it is optimal for the firm to either seed almost half of the addressable market or not seed at all.* Once $\alpha \geq \underline{\alpha}$, the firm can generate strong-enough word-of-mouth effects via sales in period 1 and it will follow a CE strategy since that way it is not losing any sales to high-type customers.

7 Comparative Statics

First, we examine the value of seeding. We start by discussing how the two seeding models (SS and CS) fare compared to the conventional CE model.

Corollary 1. *The following hold true:*
(a) $\pi^*_{SS} > \pi^*_{CE}$ *if and only if* $\alpha \in (0, \underline{\alpha})$*,*
(b) $\pi^*_{CS} > \pi^*_{CE}$ *if and only if* $\alpha \in (0, \underline{\alpha})$*.*

This corollary demonstrates that, in the absence of freemium options, a firm would benefit from seeding if the consumer priors are very small, i.e., when, at the beginning of period 1, consumers believe the software is worth less than 6.5% of its true value. In such scenarios, profit under CE is very small since the firm has to price very low in order to spur adoption and network externalities. On the other hand, under SS and CS, word-of-mouth effects are boosted by seeding, allowing the firm to price higher and obtain a higher profit.

However, as shown in the next corollary, if the firm can choose TLF, then it will never choose seeding because TLF dominates seeding in the region where, in turn, seeding dominates CE.

Corollary 2. *When* $\alpha \in (0, \underline{\alpha})$, *then* $\pi^*_{TLF} > \pi^*_{SS} = \pi^*_{CS}$.

If $\alpha < \underline{\alpha}$, seeding can only move the consumer priors upwards but still below the true software value. Uniform seeding comes at the expense of foregoing some revenue from high-type customers, thus cannibalizing some of the demand. Consequently, in the second period, the firm faces a thinned customer pool that still underestimates the product value. Therefore, clearly, TLF dominates SS. The following theorem summarizes our first set of key results, i.e., firm's optimal choice among the five business models.

Theorem 1. *From a profit perspective, for any feasible set* $\{\alpha_a, \alpha_b, \delta_b, \gamma\}$, *if the firm can choose among* CE, FLF, TLF, SS, *and* CS *business models, the following hold true:*

(a) If $0 < \alpha_a \leq \frac{\gamma+1}{2}$, *then:*

- *(i)* SS *and* CS *are always dominated strategies.*
- *(ii) If* $\delta_b \leq \frac{\alpha_a}{\gamma}$ *and* $\alpha_b \geq \frac{\gamma+1}{2\gamma} - \frac{\alpha_a}{\gamma}$, *then* CE *is the dominating strategy.*
- *(iii) If* $\delta_b \geq \frac{\alpha_a}{\gamma}$ *and* $\alpha_b + \delta_b \geq \frac{1}{2} + \frac{1}{2\gamma}$, *then* FLF *is the dominating strategy.*
- *(iv) If* $\alpha_b \leq \frac{\gamma+1}{2\gamma} - \frac{\alpha_a}{\gamma}$ *and* $\alpha_b + \delta_b \leq \frac{1}{2} + \frac{1}{2\gamma}$, *then* TLF *is the dominating strategy.*

(b) If $\frac{\gamma+1}{2} < \alpha_a$, *then:*

- *(i)* TLF, SS, *and* CS *are always dominated strategies.*
- *(ii) If* $\delta_b \leq \frac{\alpha_a}{\gamma}$, *then* CE *is the dominating strategy.*
- *(iii) If* $\delta_b \geq \frac{\alpha_a}{\gamma}$, *then* FLF *is the dominating strategy.*[5]

Figure 2 summarizes the results in Theorem 1. Each of CE, FLF, and TLF business models can dominate a particular region of the feasible parameter space, depending on consumer priors, cross-module synergies, and true value ratio between modules. Note that Theorem 1 only describes the best strategy in any given region. A complete picture of

[5] Multiple strategies can be simultaneously dominating only alongside region boundaries.

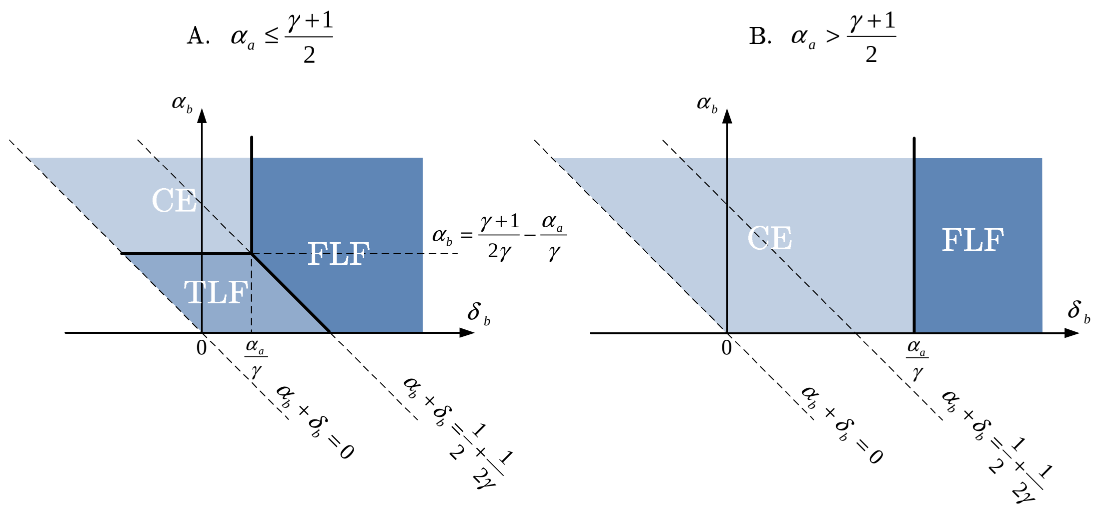

Figure 2: Firm's Optimal Strategies

the firm's ranking of these strategies by regions is included in Lemma A4, in the Online Supplement.

Whenever $\alpha_a > \gamma\delta_b$ (or, equivalently, $\alpha_a a > \delta_b b$), regardless of whether the prior on b is adjusted in the right direction or not by cross-module learning effects, model FLF is always dominated. Such an outcome may occur in a number of scenarios, including when adjustment factor δ_b is small or negative, when the true value added through model B is relatively small (i.e., small γ), or when the consumer prior on the value of the basic module is high (i.e., high α_a). Note that, in the FLF case, the freemium-generated change (increase or decrease) in the consumer valuation of module B is $\delta_b b$. Thus, if offering A for free does not increase the valuation of B by at least the consumer prior on the value of module A (i.e. $\alpha_a a$), then the firm does not find it optimal to offer A for free in both periods.

Furthermore, whenever $\alpha_a \le \frac{\gamma+1}{2}$ and $\alpha_b \le \frac{\gamma+1}{2\gamma} - \frac{\alpha_a}{\gamma}$ (or, equivalently, $\alpha \le \frac{1}{2}$), model CE is dominated. In such cases, because the overall prior on the full product is too low, it is optimal to offer a freemium strategy, thus allowing customers to adjust their priors on each module at the beginning of period 1. The choice between TLF and FLF depends on the value of $\alpha_b + \delta_b$. If δ_b is high enough, by offering module A for free the firm will elevate consumers' expectation of module B value sufficiently high to ensure optimality of FLF. Otherwise, the firm will offer the entire product for free in the first period since selling A and B together during second period under perfect information is better than selling only B for two periods under adjusted priors (imperfect information).

On the other hand, when (i) $\alpha_a \le \frac{\gamma+1}{2}$ and $\alpha_b > \frac{\gamma+1}{2\gamma} - \frac{\alpha_a}{\gamma}$, or (ii) $\alpha_a > \frac{\gamma+1}{2}$ (or, equivalently, $\alpha > \frac{1}{2}$), we see that TLF is a dominated strategy. In this region, in particular, the firm would always prefer CE over TLF since foregoing period 1 sales is suboptimal when consumers have high willingness to pay from the very beginning for the full product. This comparison follows directly from Propositions 1 and 3 since $13 - 4\sqrt{10} < \frac{1}{2}$. Furthermore, in this high α region, if δ_b is sufficiently large, CE will, in turn, be dominated by FLF.

Now that we established that, in certain scenarios, freemium models can yield a higher profit compared to the baseline model, a question of high managerial relevance is *how much more* a firm could gain by switching to a freemium model. Changes in product marketing strategy are usually a costly undertaking since they involve new advertising approaches, business process redesigning, employee re-training, etc. Therefore, such transitions make more sense when the profit improvements are of significant magnitudes. For example, when $\alpha_b + \delta_b \geq 13 - 4\sqrt{10}$ and $\alpha \geq 13 - 4\sqrt{10}$, it immediately follows that:

$$\frac{\pi^*_{FLF} - \pi^*_{CE}}{\pi^*_{CE}} = \frac{\gamma\delta_b - \alpha_a}{\gamma\alpha_b + \alpha_a} \qquad \text{and} \qquad \frac{\pi^*_{FLF} - \pi^*_{TLF}}{\pi^*_{TLF}} = \frac{\gamma(2\alpha_b + 2\delta_b - 1) - 1}{\gamma + 1}.$$

In such a case, a large δ_b value makes the FLF approach extremely desirable to the firm. Furthermore, when α_a is very small or γ is very large, the magnitude of the relative profit gain from switching from CE to FLF depends on the ratio $\frac{\delta_b}{\alpha_b}$. We can also explore other regions of the feasible parameter space for similar insights. The following corollary to Propositions 1, 2, and 3 provides insights into the asymptotic potential profit gains from switching between models:

Corollary 3. *The following hold true:*

$$(a) \qquad \lim_{\delta_b \to \infty} \frac{\pi^*_{FLF} - \pi^*_{CE}}{\pi^*_{CE}} = \lim_{\delta_b \to \infty} \frac{\pi^*_{FLF} - \pi^*_{TLF}}{\pi^*_{TLF}} = \infty, \qquad \forall\ \alpha_a, \alpha_b > 0,$$

$$(b) \qquad \lim_{\alpha_a, \alpha_b, \delta_b \to 0} \frac{\pi^*_{TLF} - \pi^*_{CE}}{\pi^*_{CE}} = \lim_{\alpha_b, \delta_b \to 0} \frac{\pi^*_{TLF} - \pi^*_{FLF}}{\pi^*_{FLF}} = \infty,$$

$$(c) \qquad \lim_{\alpha_a, \alpha_b \to 0} \frac{\pi^*_{FLF} - \pi^*_{CE}}{\pi^*_{CE}} = \lim_{\alpha_a, \alpha_b \to 0} \frac{\pi^*_{TLF} - \pi^*_{CE}}{\pi^*_{CE}} = \infty, \qquad \forall\ \delta_b > 0.$$

When consumer priors on the software modules are very small (low α_a, α_b), there is a lot to gain by switching to a freemium model, as illustrated in parts (b) and (c) of Corollary 3. The same is true when offering A for free leads to a big upwards adjustment of the consumers' valuation of B (high δ_b). Moreover, we note that even switching between two freemium models can greatly increase profitability depending on α_a, α_b, and δ_b. While Corollary 3 describes extreme cases, profit gains can be significant by switching between models even for moderate parameter values. For example, consider a case where $\alpha \in [0.4, 0.5]$, $\gamma = 4$, and $\alpha_b + \delta_b \geq 0.75$. Then $\pi^*_{CE} \leq 0.25c$, $\pi^*_{TLF} = 0.25c$, and $\pi^*_{FLF} \geq 0.3c$. By switching to

FLF, the firm can achieve at least 20% profit increase. Thus, it is important for the firm not just to consider implementing any freemium model, but to carefully choose between the available freemium options.

8 Social Welfare Analysis

It is straightforward to show that, under a central planner economy, the optimal price under each of the five models is zero due to sunk costs. In this section we focus on the more interesting social welfare implications when the firm chooses its optimal pricing strategy. We denote by SW_M^* the social welfare corresponding to the profit maximizing strategy under model M, where $M \in \{CE, FLF, TLF, SS, CS\}$. Define function $\phi : [0, 13-4\sqrt{10}] \to \mathbb{R}$, $\phi(x) = 1 - \frac{1+2x+2x^2}{2(1+x)\left(\sqrt{1+x}+\sqrt{2x}\right)^2}$. We show in the Online Supplement in Lemma A6 that ϕ is strictly increasing over the interval $[0, 13-4\sqrt{10}]$, with $\phi(0) = \frac{1}{2}$ and $\phi(13-4\sqrt{10}) \approx 0.82 < \frac{7}{8} < 1$. Then, we have the following social welfare ranking:

Theorem 2. *The following hold true:*

(a) $SW_{TLF}^* > \max\{SW_{CE}^*, SW_{SS}^*, SW_{CS}^*\}$ *for any* $\alpha > 0$*;*

(b) $SW_{FLF}^* \geq SW_{TLF}^*$ *if and only if one of the following conditions is satisfied:*

(i) $\gamma \leq \frac{1}{3}$*; or*

(ii) $\frac{1}{3} < \gamma \leq 1$ *and* $\phi^{-1}\left(\frac{7\gamma-1}{8\gamma}\right) \leq \alpha_b + \delta_b$*; or*

(iii) $1 < \gamma < \frac{1}{7-8\phi(13-4\sqrt{10})} \approx 2.262$ *and* $\phi^{-1}\left(\frac{7\gamma-1}{8\gamma}\right) \leq \alpha_b + \delta_b < 13-4\sqrt{10}$.

The results in Theorem 2 are illustrated in Figure 3. Note that, when the firm maximizes profit, social welfare under CE, SS, and CS models is always smaller than under TLF model. Thus, highest social welfare will be attained under either FLF or TLF. *In other words, freemium, in one form or another, is always preferred from the society's perspective.* The adoption dynamics for FLF can be explained through Figure 1 with $\alpha_b + \delta_b$ instead of α on the horizontal axis. When $\alpha_b + \delta_b \geq 13-4\sqrt{10}$, then, under FLF, we have only period 1 adoption and exactly half of the potential customers purchase module B. Therefore, under both FLF and TLF, high-type customers (with type $\theta \geq \frac{1}{2}$) get to use both modules A and B for two periods. The difference is that, under FLF, low-type customers ($\theta < \frac{1}{2}$) get module A for free in both periods, while, under TLF, low-type customers get both modules for free but only in the first period. Thus, in that region, $SW_{FLF}^* \geq SW_{TLF}^*$ if and only if $\gamma \leq 1$.

When $\alpha_b + \delta_b < 13-4\sqrt{10}$, we have adoption in both periods under FLF. If the

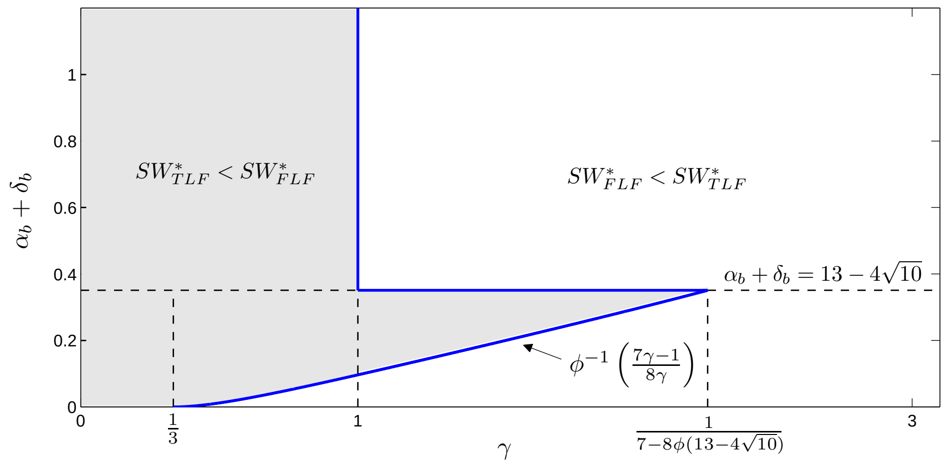

Figure 3: Social welfare ranking between TLF and FLF when the firm maximizes profit.

basic module is significantly more valuable than the premium module (i.e., $\gamma \leq \frac{1}{3}$), then FLF model generates the highest social welfare since it is more beneficial to offer the higher value module (in this case A) for free for two periods rather than give both high and low value modules for free in one period. As γ increases but stays moderate (i.e., $\frac{1}{3} \leq \gamma \leq \frac{1}{7-8\phi(13-4\sqrt{10})}$), in order for FLF to generate higher social welfare than TLF it is necessary for increasingly more adoption to happen in the first period so that more customers get benefits from using B over two periods. This, in turn, implies higher threshold values for $\alpha_b + \delta_b$, as it can be seen from Figure 1. In Theorem 2 and in Figure 3, this threshold is captured by line $\alpha_b + \delta_b = \phi^{-1}\left(\frac{7\gamma-1}{8\gamma}\right)$, which is increasing in γ.[6] Last, when $\frac{1}{7-8\phi(13-4\sqrt{10})} \leq \gamma$, then module B is significantly more valuable than module A. Under FLF, as we can see from Figure 1, even the highest level of adoption of module B in period 1, when $\alpha_b + \delta_b$ is very close to $13 - 4\sqrt{10}$, only guarantees that a small group of customers with types below $\frac{1}{2}$ get to use B over two periods and, in such a case, adoption would be very low in period 2. In such a scenario, under both FLF and TLF, high types ($\theta \geq \frac{1}{2}$) use both modules in both periods and low types ($\theta < \frac{1}{2}$) use module A for free in period 1. The difference is that, under FLF, there are a limited group of low-type consumers that use B for one or two periods, and all low-type consumers get to use A for free in period 2, while, under TLF, all low-type consumers get to use B in the first period. Given the large difference between the values of modules A and B, social welfare is maximized under TLF in this region.

For all the feasible parameter combinations, under a profit maximizing firm, we discussed in Theorems 1 and 2 which strategies yield the highest profit and the highest social welfare, respectively. We conclude this section by exploring opportunities for policy adjust-

[6]If $\frac{1}{3} \leq \gamma \leq \frac{1}{7-8\phi(13-4\sqrt{10})}$ then we have $\frac{1}{2} < \frac{7\gamma-1}{8\gamma} < \phi(13-4\sqrt{10})$ and $\phi^{-1}\left(\frac{7\gamma-1}{8\gamma}\right)$ is well defined (cases (b.ii) and (b.iii) of Theorem 2).

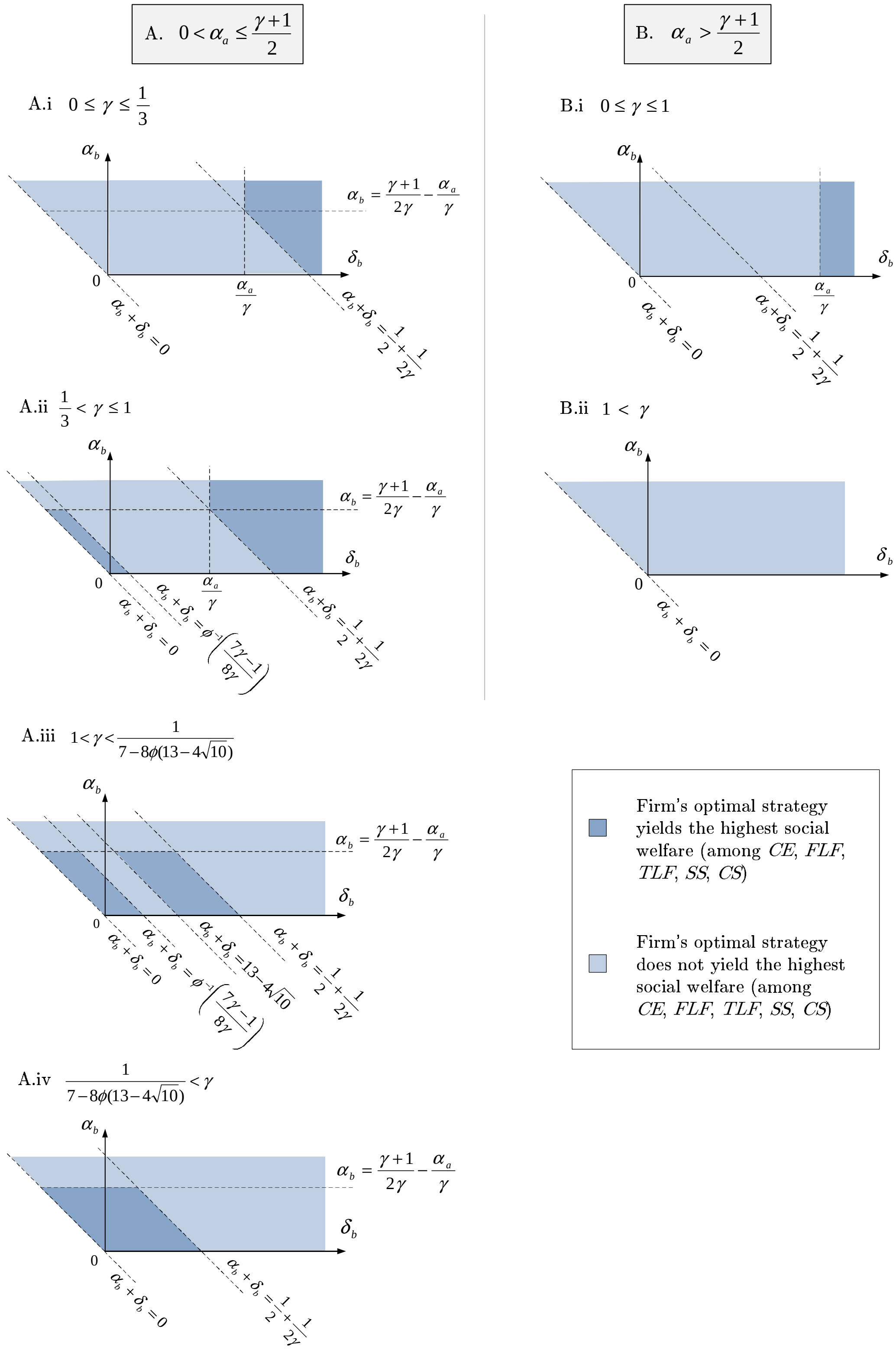

Figure 4: Plots describing when the firm's optimal strategy also yields the highest social welfare (among CE, FLF, TLF, SS, and CS).

ments (e.g., subsidies) aimed towards *aligning* the firm's and the society's interests. We depict in Figure 4 regions where a profit maximizing firm ends up choosing the strategy that actually yields the highest social welfare among CE, FLF, and TLF (we already know that SS and CS are dominated under both profit and social welfare). First, part (a) of Theorem 2 shows that it is *always* desirable from a social standpoint for the firm to switch from a baseline CE model to a freemium model. Where CE is optimal for the firm, the highest social welfare is achieved under FLF or TLF depending on the values of γ and $\alpha_b + \delta_b$, as illustrated in part (b) of Theorem 2. Second, when the firm chooses FLF, that strategy can yield the highest social welfare if and only if $\gamma \leq 1$ (panels A.i, A.ii, and B.i of Figure 4), i.e., when value added through module B is less than the value of module A. This case may correspond to scenarios where the firm offers a general multi-purpose platform product (module A) and allows users to purchase additional very specialized functionality separately (module B) which, on average, brings less benefits to users compared to the platform itself. Otherwise, social welfare can be increased by employing TLF in this region (panels A.iii, A.iv, and B.ii of Figure 4). Third, when the firm chooses TLF, that approach can be socially optimal only when $\alpha_a \leq \frac{\gamma+1}{2}$ and $\gamma > \frac{1}{3}$. When $\gamma \leq \frac{1}{3}$, FLF is socially optimal. When $\frac{1}{3} < \gamma < \frac{1}{7-8\phi(13-4\sqrt{10})}$, FLF becomes less socially desirable as firm's optimal choice (TLF) becomes socially better first for low $\alpha_b + \delta_b$ (Panel A.ii of Figure 4) and then also for high enough $\alpha_b + \delta_b$ (panel A.iii of Figure 4). Ultimately, when $\frac{1}{7-8\phi(13-4\sqrt{10})} \leq \gamma$ (panel A.iv of Figure 4), firm's optimal TLF choice is socially optimal for all $\alpha_b + \delta_b < \frac{1}{2} + \frac{1}{2\gamma}$. Last, it is particularly interesting to note that when $\alpha_a \geq \frac{\gamma+1}{2}$ and $\gamma > 1$ the firm never chooses the strategy that maximizes the social welfare (TLF). In this scenario the firm does not want under any circumstance to invest into fully informing consumers about the true value of the product by completely forfeiting period 1 sales. This, in turn, may prompt the government to subsidize the firm in order to incentivize it to switch to TLF, thus educating customers about the product benefits.

9 Value of Seeding

In this section, we explore the value of seeding. In our current setting, uniform seeding turns out to be suboptimal. Yet, as discussed in the Introduction, various forms of seeding are observed in software markets. In order to understand some of firms' incentives to use seeding, we extend our analysis by exploring how relaxing some of the assumptions of our model may reduce the performance gap between seeding and the other business models, and, perhaps, lead to seeding emerging as the optimal approach. More precisely, we look at the informativeness of word of mouth signals, the cost structure, and the prospect of targeted seeding.

9.1 Degree of Informativeness of Market Signals

So far we explored market dynamics assuming an intermediate degree of informativeness of signals. When the signals are uninformative ($w \approx 0$), then seeding carries little value since giving the product for free to some customers will not allow the firm to charge others more. When the signals are highly informative ($w \to \infty$), then potential customers can learn the true valuation from a very small mass of existing adopters. In such a case, when customers severely underestimate the overall value of the product ($\alpha < \frac{1}{4}$) and giving module A for free does not induce consumers to value module B at a high enough level either ($\alpha_b + \delta_b < \frac{1}{4}$), then it can be shown (proof included in the Online Supplement) that:

$$\max\left\{\lim_{w\to\infty}\pi^*_{CE}, \lim_{w\to\infty}\pi^*_{FLF}\right\} < \frac{c}{4} = \pi^*_{TLF} = \lim_{w\to\infty}\pi^*_{SS} = \lim_{w\to\infty}\pi^*_{CS}.$$

The intuition is rather simple. Under SS and CS seeding models, the firm may reap the benefits of highly informative signals by seeding a negligible portion of the relevant market and then pricing high, selling only in the second period (when the consumers would be informed). On the other hand, under CE and FLF, the firm needs to sell in the first period in order to jump-start word-of-mouth effects, which imposes an upper bound on prices given consumer priors. If consumers underestimate a lot the value of the software, then the firm must price low so that adoption gets started, which in turn, precludes it from reaping high profits in the second period. In this case, while not dominating, SS and CS can yield profits that are within negligible range from the optimal profit.

9.2 Cost Structure

Our model assumes negligible operational costs for all the business strategies. Results can be extended to non-negligible but comparable costs without changes to the analysis. However, if firm costs differ significantly across models, results might change. In the case of FLF or TLF, if the market is large and split into consumer clusters that are not heavily interconnected, unless the firm extensively advertises the free option as well, many consumers may not be aware in the beginning about the freemium structure of the offer, and, as such, may not update their priors. This phenomenon of delay in market information dissemination has been treated theoretically (e.g., Kalish 1985, Mankiw and Reis 2002), and advertising is recognized as one way to reduce this delay. However, if freemium advertising costs far exceed the seeding costs, then, when customers heavily underestimate the modules and the cross-module signal δ_b is not very informative, uniform seeding may become optimal.

Also, at the individual level, some consumers may be reluctant to invest resources to install and test the time-limited free trial since they know that, unless they buy the license for the product, the installation will lock itself in the near future (Anderson 2009). This further diminishes the ability of the firm to use the TLF approach to achieve market-wide awareness regarding the value of the software. On the other hand, if customers were seeded licensed copies of the software with no attached expiration clock, then they may be more inclined to use it.

9.3 Targeted versus Uniform Seeding

We also assumed in the paper that word-of-mouth effects are homogeneous in strength regardless of the originating consumers who adopted before. Furthermore, we assumed that the firm knows the distribution of consumer types but does not know the precise type of any given consumer. In that case, uniform (random) seeding is the only available seeding approach.

However, if (i) there are opinion-leaders or experts that have a considerably stronger influence on their peers or better skills to comprehensively explore the functionality of the software compared to average reviewers, and (ii) the firm can identify such consumers, then the firm might find it optimal to seed them first in order to generate stronger buzz about the product. This seems to be happening to a certain extent in software markets. For example, software companies occasionally showcase their products at developer conferences or conventions, distributing free samples to members of a highly informed audience that attends such events. There is a higher than random likelihood that such users are interested in the software and that they are going to explore its functionality in more depth than casual users, thus offering more insightful comments on the performance and features of the product to the rest of the community. Similar to the argument in §9.1, if some of the opinion-leaders or experts are extremely influential, when consumers severely underestimate the module values, then the firm may seed a few of those influencers and price high in order to reap the benefits in the second period. Thus, SS and CS will be getting closer to TLF in terms of performance. On the other hand, again, under CE or FLF, the firm must keep the prices low to start adoption in period 1, which diminishes its revenue in the second period.

Note that the above argument does not include third-party product review sites such as CNET.com since firms may submit their products to such sites for evaluation regardless of the chosen business model as part of the initial advertising strategy when consumers undervalue the product. In that sense, a professional review of the product on one of those sites may be considered to help in the formation of the initial priors a_0 and b_0.

10 Conclusions

In this paper we model analytically the impact of free consumption offerings on consumer valuation learning. We provide an extensive head-on comparison between the conventional charge-for-everything business model and several freemium and seeding models extant in software markets. Our models are based on an integrated framework capturing multi-period adoption dynamics, network externalities, customer valuation learning, and software modularity. This allows us to conduct previously unexplored comprehensive performance benchmarking among models and to derive policy implications. Moreover, our analysis sheds more light on various marketing approaches employed in the software industry. In our setting, under intermediate informativeness of word-of-mouth signals, CE, TLF, and FLF can each be a dominating strategy for the firm, depending on consumer priors on the value of individual software modules, perceptions of cross-module synergies, and overall value distribution across modules, whereas uniform seeding is always dominated by either freemium models or conventional for-fee models. Our analysis also offers relevant managerial insights, as we show that switching from CE model to freemium models can dramatically increase profitability, as can switching between different freemium models. Furthermore, we show that freemium is always preferred from the society's perspective, and derive recommendations as to when the firm needs to be subsidized in order for the social welfare to be increased. Finally, we highlight the value of seeding under a more relaxed set of assumptions. Among others, we show that seeding may become optimal under high advertising costs and very informative or heterogeneous word-of-mouth signals.

Given that the goal of this study is to seek insights by comparing and contrasting several established business models, our framework was stylized for tractability. This presents opportunities for our analysis to be extended in multiple ways, perhaps using simulations or other numerical approaches when closed-form solutions like ours cannot be derived. We already discussed some of these extensions in §9. In addition, throughout this study, we focussed on software products characterized by one-time purchase, where the users run the software off their own machines and the developer does not invest additional resources to support the consumption of the product. For simplicity we do not consider quality improvement and maintenance via patching. However, as discussed in the Introduction, freemium applicability extends beyond such products, in particular branching into the rapidly growing markets for software-as-a-service products, where adoption dynamics are slightly different and the revenue model is in many cases subscription-based, involving recurring payments from the installed base. In such markets, service providers may incur non-negligible operational costs associated with running the application (on own hardware infrastructure or on stable environments sourced from platform-as-a-service providers such as Salesforce.com or infrastructure-as-a-service providers such as Amazon or Rackspace). Offering a feature-limited freemium model involves bleeding costs from supporting the service for a mass of non-paying customers for the entire product lifecycle, posing challenges to the viability of the model (Sixteen Ventures 2010). Sometimes, service providers cover a portion of these bleeding costs by offering an ad-supported model. It would be very interesting to explore the conditions under which the freemium models would dominate CE models in the software-as-a-service markets.

Another extension would be to consider dynamic pricing, along with the presence of strategic customers. At the beginning of period 1, strategic customers might consider delaying adoption until period 2 if they anticipate a decrease in price. By contrast, we assume the firm commits to fixed pricing. While our two-period models (CE, FLF or CS) predict like behavior whereby customers may balk early but adopt later in the second period, the reasons are quite different. This behavior does not emerge because of strategic choice at the beginning of period 1, but because of consumer valuation learning at the end of period 1 attributed to word-of-mouth effects.

Also, it would be very interesting to see how hybrid freemium models (described in the Introduction) fare compared to pure ones. While beyond the scope of this paper, network effects could also be incorporated at the utility level in addition to the network externalities governing the word-of-mouth effect on valuation learning. However, in our current setting, adding these effects would make the model intractable. Last, it would be intriguing to complement our analytical study with empirical testings as well as laboratory and field experiments that would explore consumer valuation learning in software markets.

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Online Supplement for Manuscript

"When Should Software Firms Commercialize New Products via Freemium Business Models?"

- Proofs of Results -

Proof of Lemma 1. Follows immediately from the equilibrium discussion in §4. □

Proof of Lemma 2. Based on Lemma 1, we know that $\theta_2 < \theta_1$ if and only if $c_2 > 2c_1$. Given that, under constraint (4), $N_1 = 1-\theta_1 = 1-\frac{p}{2c\alpha}$, we have $c_2 = c\alpha+\left(1-\frac{p}{2c\alpha}\right)(c-c\alpha) = c - \frac{p(1-\alpha)}{2\alpha}$. Therefore, by simple manipulation, we obtain:

$$c_2 > 2c_1 \;\Leftrightarrow\; c(1-2\alpha) > \frac{p(1-\alpha)}{2\alpha}. \tag{A.1}$$

Now we move ahead to prove the result in Lemma 2.

Direction '⇒'. Suppose adoption extends to period 2. Then, obviously, feasibility constraint in equation (4) must be satisfied. We have $c_2 > 2c_1$. Note that, as discussed in §4, when $\alpha \geq 1$ consumers never adopt in period 2 (they either adopt in period 1 and own the product in period 2 or do not adopt at all). Thus, if they adopt in period 2, it must be the case that $\alpha < 1$. From equation (A.1), it immediately follows that $\alpha < \frac{1}{2}$. Rewriting equation (A.1) immediately yields $p < \frac{2\alpha c(1-2\alpha)}{1-\alpha} < 2c\alpha$.

Direction '⇐'. Result follows immediately from the conditions. First, since $0 < \frac{1-2\alpha}{1-\alpha} < 1$, implicitly the price feasibility constraint in equation (4) is satisfied. Next, it is easy to see that (A.1) is satisfied, i.e., $c_2 > 2c_1$. Then, we immediately have $\theta_2 < \theta_1$. □

Proof of Proposition 1. The CE model is a particular case of the CS model with $k = 0$. We refer readers to the proof of Lemma 4 for the complete solution.

For the social welfare analysis, we have two cases. When $13 - 4\sqrt{10} \leq \alpha$, we only have period 1 adoption and it immediately follows that $SW^*_{CE} = \frac{3c}{4}$. When, $0 < \alpha < 13 - 4\sqrt{10}$, we have adoption in both periods, and we obtain $SW^*_{CE} = c\left[1 - \frac{(1+2\alpha+2\alpha^2)}{2(1+\alpha)\left(\sqrt{1+\alpha}+\sqrt{2\alpha}\right)^2}\right]$. □

Proof of Proposition 2. The analysis is similar to that in CE model, using the substitution argument immediately preceding Proposition 2 in the main text. □

Proof of Proposition 3. In this case, we are looking at adoption solely in period 2 based on $c_2 = c$. Thus $\theta_2 = \frac{p}{c}$ and $\pi_{TLF}(p) = p\left(1-\frac{p}{c}\right)$. It immediately follows that $p^*_{TLF} = \frac{c}{2}$ and $\pi^*_{TLF} = \frac{c}{4}$. By taking into account that the product is offered for free during period 1, we obtain $SW^*_{TLF} = \int_0^1 c\theta d\theta + \int_{\frac{1}{2}}^1 c\theta d\theta = \frac{7c}{8}$. □

Proof of Proposition 4. We first optimize on price, fixing seeding rate k. We obtain:

$$p^*_{SS}(k) = \frac{c[\alpha + k(1-\alpha)]}{2} \qquad \text{and} \qquad \pi_{SS}(p^*_{SS}(k), k) = \frac{c(1-k)[\alpha + k(1-\alpha)]}{4}.$$

Next, by optimizing on k, we obtain

- If $0 < \alpha < \frac{1}{2}$, then $k^*_{SS} = \frac{1-2\alpha}{2(1-\alpha)}$, $p^*_{SS} = \frac{c}{4}$, $\pi^*_{SS} = \frac{c}{16(1-\alpha)}$.
- If $\frac{1}{2} < \alpha < 1$, then $k^*_{SS} = 0$, $p^*_{SS} = \frac{c\alpha}{2}$, $\pi^*_{SS} = \frac{c\alpha}{4}$.

For any given $k \in [0, 1)$, it can be shown that $SW^*_{SS}(k) = \frac{3c}{8} + \frac{5ck}{8}$. Using optimal values k^*_{SS}, we obtain the results. □

Proof of Lemma 3. Direction '⇒'. Suppose adoption extends to period 2. Constraint (11) must be satisfied. Note that, similar to the CE case, when $\alpha \geq 1$ consumers never adopt in period 2 (they either adopt in period 1 and own the product in period 2 or do not adopt at all). This is because overestimation induces a downwards adjustment of the consumers' willingness to pay. Thus, if they adopt in period 2, it must be the case that $\alpha < 1$. We consider two cases:

(a) If $\frac{k}{1+k} \leq \alpha$, then we have $2c\alpha \geq c[\alpha+(1-\alpha)k]$. Therefore, constraint (11) translates into $p < 2c\alpha$. Under this constraint, adoption will start in period 1 and we have $\theta_1 = \frac{p}{2c\alpha}$. Given that $N_1 = k+(1-k)(1-\theta_1) = k+(1-k)\left(1 - \frac{p}{2c\alpha}\right) = 1-\frac{p(1-k)}{2c\alpha}$, we immediately obtain $c_2 = c - \frac{p(1-k)(1-\alpha)}{2\alpha}$.

In this case, we know that $\theta_2 < \theta_1$ if and only if $c_2 > 2c_1$.

$$c_2 > 2c_1 \;\Leftrightarrow\; c(1-2\alpha) > \frac{p(1-k)(1-\alpha)}{2\alpha}. \tag{A.2}$$

Given that $k \in [0, 1)$ and, as mentioned above, we only consider the case $\alpha < 1$, from equation (A.2) we have $c(1-2\alpha) > 0$, or $\alpha < \frac{1}{2}$. Next, by rewriting equation (A.2), we obtain $p < \frac{2\alpha c(1-2\alpha)}{(1-\alpha)(1-k)}$.

(b) If $0 < \alpha < \frac{k}{1+k}$, then $0 < 2c\alpha < c[\alpha + (1-\alpha)k]$, and constraint (11) translates into $p < c[\alpha + k(1-\alpha)]$. We have two subcases:

(i) $p < 2c\alpha$. In this case, adoption starts in period 1. Similar to case (a), adoption in period 2 yields constraints $\alpha < \frac{1}{2}$ and $p < \frac{2\alpha c(1-2\alpha)}{(1-\alpha)(1-k)}$. However, since $\frac{k}{1+k} < \frac{1}{2}$ and $\alpha < \frac{k}{1+k}$, we have $1 - 2\alpha \geq (1-\alpha)(1-k)$ and period 2 adoption constraints are automatically satisfied.

(ii) $2c\alpha \leq p < c[\alpha+k(1-\alpha)]$. In this case, there is no adoption in period 1 (negligible adoption of mass 0 happens when $p = 2c\alpha$); only in period 2. $c_2 = c[\alpha+k(1-\alpha)] \geq p$. Thus, $\theta_2 = \frac{p}{c[\alpha+k(1-\alpha)]} < 1 = \theta_1$. No additional constrains are necessary for adoption to occur in the second period.

Direction '⇐'. Follows immediately in each case from the conditions. □

Lemma A1. *Consider a fixed value $k \in (0,1)$. Let us define the function:*

$$h_k(\alpha) = [\alpha + (1-\alpha)k]\left(\sqrt{2\alpha + (1-\alpha)(1-k)} + \sqrt{2\alpha}\right)^2 - 8\alpha.$$

Then, over the interval $\left[\frac{-1-k^2+\sqrt{1+3k^2}}{1-k^2}, \frac{k}{3+k}\right]$, *$h_k(\alpha)$ is strictly decreasing in α and equation $h_k(\alpha) = 0$ has a unique solution $\tilde{\alpha}(k)$.*

Proof. Differentiating $h_k(\alpha)$ we obtain: we have:

$$\begin{aligned} \frac{\partial h_k}{\partial \alpha} &= (1-k)\left(\alpha(3+k) - k + 1 + 2\sqrt{2\alpha}\sqrt{2\alpha + (1-\alpha)(1-k)}\right) \\ &\quad + (\alpha + k(1-\alpha))\left(3 + k + \frac{4\alpha(1+k) + 2(1-k)}{\sqrt{2\alpha}\sqrt{2\alpha + (1-\alpha)(1-k)}}\right) - 8. \end{aligned} \tag{A.3}$$

Consider $\alpha \in \left[\frac{-1-k^2+\sqrt{1+3k^2}}{1-k^2}, \frac{k}{3+k}\right]$, it can be shown that:

$$\alpha(3+k) - k + 1 + 2\sqrt{2\alpha}\sqrt{2\alpha + (1-\alpha)(1-k)} < 3, \tag{A.4}$$

$$(\alpha + k(1-\alpha))(3+k) \leq 4k, \tag{A.5}$$

$$\frac{(\alpha + k(1-\alpha))(4\alpha(1+k) + 2(1-k))}{\sqrt{2\alpha}\sqrt{2\alpha + (1-\alpha)(1-k)}} < 4. \tag{A.6}$$

Putting together (A.3), (A.4), (A.5), and (A.6), we obtain:

$$\frac{\partial h_k}{\partial \alpha} < 3(1-k) + 4k + 4 - 8 = k - 1 < 0, \ \forall \alpha \in \left[\frac{-1-k^2+\sqrt{1+3k^2}}{1-k^2}, \frac{k}{3+k}\right]. \tag{A.7}$$

We show in the proof of Lemma 4 (to follow) that $h_k(\frac{k}{3+k}) \leq 0$ and $h_k\left(\frac{-1-k^2+\sqrt{1+3k^2}}{1-k^2}\right) \geq 0$. Therefore equation $h_k(\alpha) = 0$ has a unique solution $\tilde{\alpha}(k)$ in this interval. □

Proof of Lemma 4. We first consider $k \in (0,1)$. We are looking at multiple cases.

Case 1. $\frac{1}{2} \leq \alpha$.

In this case, as per Lemma 3, there is no additional adoption in period 2. Thus, $\pi = p(N_1 - k) = p(1-k)\left(1 - \frac{p}{2c\alpha}\right)$. It follows that $p^* = c\alpha$ and $\pi(p^*) = \frac{c\alpha(1-k)}{2}$.

Case 2. $\frac{k}{1+k} \leq \alpha < \frac{1}{2}$. We consider two subcases:

a. $p \geq \frac{2c\alpha(1-2\alpha)}{(1-\alpha)(1-k)}$. In this case, as per Lemma 3, there is no adoption in period 2 and profit functions is $\pi = p(N_1 - k) = p(1-k)\left(1 - \frac{p}{2c\alpha}\right)$. Solving under constraints $\alpha < \frac{1}{2}$ and $p \geq \frac{2c\alpha(1-2\alpha)}{(1-\alpha)(1-k)}$ we obtain:

$$p_a^* = \begin{cases} c\alpha, & \text{if } \frac{1+k}{3+k} < \alpha < \frac{1}{2}, \\ \frac{2c\alpha(1-2\alpha)}{(1-\alpha)(1-k)}, & \text{if } \frac{k}{1+k} \leq \alpha \leq \frac{1+k}{3+k}. \end{cases} \tag{A.8}$$

$$\pi(p_a^*) \quad = \quad \begin{cases} \frac{c\alpha(1-k)}{2} & , \text{ if } \frac{1+k}{3+k} < \alpha < \frac{1}{2}, \\ \frac{2c\alpha(1-2\alpha)(\alpha+\alpha k-k)}{(1-k)(1-\alpha)^2} & , \text{ if } \frac{k}{1+k} \le \alpha \le \frac{1+k}{3+k}. \end{cases} \tag{A.9}$$

b. $p \le \frac{2c\alpha(1-2\alpha)}{(1-\alpha)(1-k)}$. When $p < \frac{2c\alpha(1-2\alpha)}{(1-\alpha)(1-k)}$, according to Lemma 3, we have adoption in both periods. When $p = \frac{2c\alpha(1-2\alpha)}{(1-\alpha)(1-k)}$, we can consider that adoption with mass 0 occurs in period 2. Profit is given by:

$$\pi = p(N_2 - k) = p(1-k)\left(1 - \frac{p}{c_2}\right) = p(1-k)\left(1 - \frac{p}{c - \frac{p(1-\alpha)(1-k)}{2\alpha}}\right). \tag{A.10}$$

The solutions $p_1 < p_2$ to the equation $\frac{\partial \pi}{\partial p}(p) = 0$ are:

$$p_{1,2} \quad = \quad \frac{2c\alpha}{(1-\alpha)(1-k)}\left(1 \pm \sqrt{\frac{2\alpha}{2\alpha + (1-\alpha)(1-k)}}\right) > 0, \tag{A.11}$$

and we have:

$$\frac{\partial \pi}{\partial p} \begin{cases} > 0 & , \text{ if } p < p_1, \\ = 0 & , \text{ if } p = p_1, \\ < 0 & , \text{ if } p_1 < p < p_2, \\ = 0 & , \text{ if } p = p_2, \\ > 0 & , \text{ if } p > p_2. \end{cases}$$

Therefore, $\pi(p)$ is increasing over $[0, p_1]$, decreasing over $[p_1, p_2]$ and increasing over $[p_2, \infty)$. It is straightforward to see that p_2 is infeasibile under constraint (11) because $2c\alpha = \max\{2c\alpha, c[\alpha + k(1-\alpha)]\} < p_2$. On the interval $\left[0, \frac{2c\alpha(1-2\alpha)}{(1-\alpha)(1-k)}\right]$, $\pi(p)$ is either *increasing* or *unimodal* with peak p_1. Solving under constraints $\frac{k}{1+k} \le \alpha < \frac{1}{2}$ and $p \le \frac{2c\alpha(1-2\alpha)}{(1-\alpha)(1-k)}$, we obtain the following solution:

$$p_b^* = \begin{cases} \frac{2c\alpha(1-2\alpha)}{(1-\alpha)(1-k)} & , \frac{1}{\sqrt{3+k^2}+1-k} < \alpha < \frac{1}{2}, \\ p_1 = \frac{2c\alpha}{(1-\alpha)(1-k)}\left(1 - \sqrt{\frac{2\alpha}{2\alpha+(1-\alpha)(1-k)}}\right) & , \frac{k}{1+k} \le \alpha \le \frac{1}{\sqrt{3+k^2}+1-k}, \end{cases} \tag{A.12}$$

$$\pi(p_b^*) = \begin{cases} \frac{2c\alpha(1-2\alpha)(\alpha+\alpha k-k)}{(1-k)(1-\alpha)^2} & , \frac{1}{\sqrt{3+k^2}+1-k} < \alpha < \frac{1}{2}, \\ \frac{2c\alpha(\sqrt{2\alpha+(1-k)(1-\alpha)}-\sqrt{2\alpha})^2}{(1-k)(1-\alpha)^2} & , \frac{k}{1+k} \le \alpha \le \frac{1}{\sqrt{3+k^2}+1-k}. \end{cases} \tag{A.13}$$

The formula for the second case ($\frac{k}{1+k} \le \alpha \le \frac{1}{\sqrt{3+k^2}+1-k}$) is derived using the fact that $(1-\alpha)(1-k) = (\sqrt{2\alpha + (1-\alpha(1-k)} - \sqrt{2\alpha})(\sqrt{2\alpha + (1-\alpha(1-k)} + \sqrt{2\alpha})$.

Reconciling subcases 2.a and 2.b.

We reconcile the above subcases, characterizing the *choice* of the firm as whether to allow adoption in second period or not.

(i) $\frac{k}{1+k} \le \alpha \le \frac{1+k}{3+k}$. Since $k < 1$, it immediately follows that:

$$\pi(p_a^*) = \frac{2c\alpha(1-2\alpha)(\alpha+\alpha k-k)}{(1-k)(1-\alpha)^2} < \frac{2c\alpha(\sqrt{2\alpha+(1-\alpha)(1-k)}-\sqrt{2\alpha})^2}{(1-k)(1-\alpha)^2} = \pi(p_b^*).$$

(ii) $\frac{1+k}{3+k} < \alpha < \frac{13+2k+k^2-4\sqrt{10+2k^2+4k}}{(1-k)^2}$. Then, it can be shown that:

$$\pi(p_a^*) = \frac{c\alpha(1-k)}{2} < \frac{2c\alpha(\sqrt{2\alpha+(1-\alpha)(1-k)}-\sqrt{2\alpha})^2}{(1-k)(1-\alpha)^2} = \pi(p_b^*).$$

(iii) $\alpha = \frac{13+2k+k^2-4\sqrt{10+2k^2+4k}}{(1-k)^2}$. Then, it can be shown that:

$$\pi(p_a^*) = \frac{c\alpha(1-k)}{2} = \frac{2c\alpha(\sqrt{2\alpha+(1-\alpha)(1-k)}-\sqrt{2\alpha})^2}{(1-k)(1-\alpha)^2} = \pi(p_b^*).$$

Again, we assume the firm prefers earlier revenues to later revenues. Therefore, in this case, the firm will price so that there is no adoption in period 2.

(iv) $\frac{13+2k+k^2-4\sqrt{10+2k^2+4k}}{(1-k)^2} < \alpha \le \frac{1}{\sqrt{3+k^2}+1-k}$. It can be shown that:

$$\pi(p_a^*) = \frac{c\alpha(1-k)}{2} > \frac{2c\alpha(\sqrt{2\alpha+(1-\alpha)(1-k)}-\sqrt{2\alpha})^2}{(1-k)(1-\alpha)^2} = \pi(p_b^*).$$

(v) $\frac{1}{\sqrt{3+k^2}+1-k} < \alpha < \frac{1}{2}$. Then, it can be shown that:

$$\pi(p_a^*) = \frac{c\alpha(1-k)}{2} > \frac{2c\alpha(1-2\alpha)(\alpha+\alpha k-k)}{(1-k)(1-\alpha)^2} = \pi(p_b^*).$$

Case 3. $0 < \alpha < \frac{k}{1+k}$. This case exists only when $k > 0$. Then, the feasible pricing constraint (11) becomes $0 < p < c[\alpha + (1-\alpha)k]$. We discuss two subcases:

a. $2c\alpha \le p < c[\alpha + (1-\alpha)k]$. In this case, adoption occurs only in period 2 and $\pi = p(N_2 - k) = p(1-k)\left(1 - \frac{p}{c[\alpha+(1-\alpha)k]}\right)$. Maximizing profit under constraint $2c\alpha \le p < c[\alpha + (1-\alpha)k]$, we obtain:

$$p_c^* = \begin{cases} 2c\alpha & \text{, when } \frac{k}{3+k} \le \alpha < \frac{k}{1+k}, \\ \frac{c[\alpha+(1-\alpha)k]}{2} & \text{, when } 0 < \alpha \le \frac{k}{3+k}, \end{cases} \tag{A.14}$$

$$\pi(p_c^*) = \begin{cases} (1-k)2c\alpha\left(1 - \frac{2\alpha}{\alpha+(1-\alpha)k}\right) & \text{, when } \frac{k}{3+k} \le \alpha < \frac{k}{1+k}, \\ \frac{(1-k)c[\alpha+(1-\alpha)k]}{4} & \text{, when } 0 < \alpha \le \frac{k}{3+k}. \end{cases} \tag{A.15}$$

b. $0 \le p \le 2c\alpha < c[\alpha + (1-\alpha)k]$. In this case adoption occurs in both periods. Similar to the analysis in case 2.b, we obtain the same solutions $p_1 < p_2$ described in (A.11) for the equation $\frac{\partial \pi}{\partial p} = 0$. Again, p_2 is infeasibile. Then it can be shown that:

$$p_d^* = \begin{cases} \frac{2c\alpha}{(1-\alpha)(1-k)}\left(1 - \sqrt{\frac{2\alpha}{2\alpha+(1-\alpha)(1-k)}}\right) & , \frac{-1-k^2+\sqrt{1+3k^2}}{1-k^2} \le \alpha < \frac{k}{1+k}, \\ 2c\alpha & , 0 < \alpha < \frac{-1-k^2+\sqrt{1+3k^2}}{1-k^2}, \end{cases} \tag{A.16}$$

$$\pi(p_d^*) = \begin{cases} \frac{2c\alpha(\sqrt{2\alpha+(1-k)(1-\alpha)}-\sqrt{2\alpha})^2}{(1-k)(1-\alpha)^2} & , \frac{-1-k^2+\sqrt{1+3k^2}}{1-k^2} \le \alpha < \frac{k}{1+k}, \\ (1-k)2c\alpha\left(1 - \frac{2\alpha}{\alpha+(1-\alpha)k}\right) & , 0 < \alpha < \frac{-1-k^2+\sqrt{1+3k^2}}{1-k^2}. \end{cases} \tag{A.17}$$

<u>Reconciling subcases 3.a and 3.b.</u>

First, it can be shown that $\frac{k}{3+k} > \frac{-1-k^2+\sqrt{1+3k^2}}{1-k^2}$ when $k < 1$.

(i) $0 < \alpha < \frac{-1-k^2+\sqrt{1+3k^2}}{1-k^2}$. Then:

$$\pi(p_c^*) = \frac{(1-k)c[\alpha+(1-\alpha)k]}{4} > 2c\alpha(1-k)\left(1 - \frac{2\alpha}{\alpha+(1-\alpha)k}\right) = \pi(p_d^*).$$

(ii) $\frac{-1-k^2+\sqrt{1+3k^2}}{1-k^2} \le \alpha \le \frac{k}{3+k}$. In this case, we know that:

$$\pi(p_c^*) = \frac{(1-k)c[\alpha+(1-\alpha)k]}{4} \quad \text{and} \quad \pi(p_d^*) = \frac{2c\alpha(\sqrt{2\alpha+(1-k)(1-\alpha)}-\sqrt{2\alpha})^2}{(1-k)(1-\alpha)^2}.$$

Then:

$$\pi(p_c^*) \ge \pi(p_d^*) \ \Leftrightarrow h_k(\alpha) \ge 0, \tag{A.18}$$

where function $h_k(\alpha)$ was defined in Lemma A1. From Lemma A1, we know that h_k is strictly decreasing and can change sign at most once on the interval $\left[\frac{-1-k^2+\sqrt{1+3k^2}}{1-k^2}, \frac{k}{3+k}\right]$. When $\alpha = \frac{-1-k^2+\sqrt{1+3k^2}}{1-k^2}$, then $p_d^* = 2c\alpha$ and profit $\pi(p_d^*)$ is:

$$\pi(p_d^*) = \frac{2c\alpha(\sqrt{2\alpha+(1-k)(1-\alpha)}-\sqrt{2\alpha})^2}{(1-k)(1-\alpha)^2} = (1-k)2c\alpha\left(1 - \frac{2\alpha}{\alpha+(1-\alpha)k}\right).$$

It can shown that:

$$\pi(p_c^*) = \frac{(1-k)c[\alpha+(1-\alpha)k]}{4} \ge 2c\alpha(1-k)\left(1 - \frac{2\alpha}{\alpha+(1-\alpha)k}\right) = \pi(p_d^*).$$

Therefore, $h_k\left(\frac{-1-k^2+\sqrt{1+3k^2}}{1-k^2}\right) \ge 0$.

When $\alpha = \frac{k}{3+k}$, then $p_c^* = 2c\alpha$ and profit $\pi(p_c^*)$ is:

$$\pi(p_c^*) = (1-k)2c\alpha\left(1 - \frac{2\alpha}{\alpha+(1-\alpha)k}\right) = \frac{(1-k)c[\alpha+(1-\alpha)k]}{4}.$$

	Firm		Consumer Adoption Pattern
	$p^*_{CS}(k)$	$\pi^*_{CS}(k)$	
$\frac{1}{2} \leq \alpha$	$c\alpha$	$\frac{c\alpha(1-k)}{2}$	Per. 1
$\frac{1}{\sqrt{3+k^2}+1-k} < \alpha < \frac{1}{2}$	$c\alpha$	$\frac{c\alpha(1-k)}{2}$	Per. 1
$\frac{13+2k+k^2-4\sqrt{10+2k^2+4k}}{(1-k)^2} \leq \alpha \leq \frac{1}{\sqrt{3+k^2}+1-k}$	$c\alpha$	$\frac{c\alpha(1-k)}{2}$	Per. 1
$\frac{1+k}{3+k} < \alpha < \frac{13+2k+k^2-4\sqrt{10+2k^2+4k}}{(1-k)^2}$	$\frac{2c\alpha}{(1-\alpha)(1-k)}\left(1-\sqrt{\frac{2\alpha}{2\alpha+(1-\alpha)(1-k)}}\right)$	$\frac{2c\alpha(1-k)}{(\sqrt{2\alpha+(1-k)(1-\alpha)}+\sqrt{2\alpha})^2}$	Per. 1,2
$\frac{k}{1+k} \leq \alpha \leq \frac{1+k}{3+k}$	$\frac{2c\alpha}{(1-\alpha)(1-k)}\left(1-\sqrt{\frac{2\alpha}{2\alpha+(1-\alpha)(1-k)}}\right)$	$\frac{2c\alpha(1-k)}{(\sqrt{2\alpha+(1-k)(1-\alpha)}+\sqrt{2\alpha})^2}$	Per. 1,2
$\frac{k}{3+k} < \alpha < \frac{k}{1+k}$	$\frac{2c\alpha}{(1-\alpha)(1-k)}\left(1-\sqrt{\frac{2\alpha}{2\alpha+(1-\alpha)(1-k)}}\right)$	$\frac{2c\alpha(1-k)}{(\sqrt{2\alpha+(1-k)(1-\alpha)}+\sqrt{2\alpha})^2}$	Per. 1,2
$\tilde{\alpha}(k) \leq \alpha \leq \frac{k}{3+k}$	$\frac{2c\alpha}{(1-\alpha)(1-k)}\left(1-\sqrt{\frac{2\alpha}{2\alpha+(1-\alpha)(1-k)}}\right)$	$\frac{2c\alpha(1-k)}{(\sqrt{2\alpha+(1-k)(1-\alpha)}+\sqrt{2\alpha})^2}$	Per. 1, 2
$\frac{-1-k^2+\sqrt{1+3k^2}}{1-k^2} \leq \alpha < \tilde{\alpha}(k)$	$\frac{c[\alpha+(1-\alpha)k]}{2}$	$\frac{(1-k)c[\alpha+(1-\alpha)k]}{4}$	Per. 2
$0 < \alpha < \frac{-1-k^2+\sqrt{1+3k^2}}{1-k^2}$	$\frac{c[\alpha+(1-\alpha)k]}{2}$	$\frac{(1-k)c[\alpha+(1-\alpha)k]}{4}$	Per. 2

Table A1: Optimal Price - CS Model

It can be shown that:

$$\pi(p_c^*) = (1-k)2c\alpha\left(1 - \frac{2\alpha}{\alpha+(1-\alpha)k}\right) \leq \frac{2c\alpha(\sqrt{2\alpha+(1-k)(1-\alpha)} - \sqrt{2\alpha})^2}{(1-k)(1-\alpha)^2} = \pi(p_d^*).$$

Therefore, $h_k\left(\frac{k}{3+k}\right) \leq 0$.

Since h_k is strictly decreasing over the interval $\left[\frac{-1-k^2+\sqrt{1+3k^2}}{1-k^2}, \frac{k}{3+k}\right]$, then the equation $h_k(\alpha) = 0$ has a unique solution $\tilde{\alpha}(k)$ in this interval. Consequently:

$$\begin{aligned} \pi(p_c^*) > \pi(p_d^*) &\quad \text{when} \quad \frac{-1-k^2+\sqrt{1+3k^2}}{1-k^2} \leq \alpha < \tilde{\alpha}(k), \\ \pi(p_c^*) = \pi(p_d^*) &\quad \text{when} \quad \alpha = \tilde{\alpha}(k), \\ \pi(p_c^*) < \pi(p_d^*) &\quad \text{when} \quad \tilde{\alpha}(k) < \alpha \leq \frac{k}{3+k}. \end{aligned}$$

When $\alpha = \tilde{\alpha}(k)$, similar as before, we assume the firm prefers earlier revenues more, and, as such, will price so that adoption happens in both period 1 and period 2.

(iii) $\frac{k}{3+k} < \alpha < \frac{k}{1+k}$. Then, it can be shown that:

$$\pi(p_c^*) = (1-k)2c\alpha\left(1 - \frac{2\alpha}{\alpha+(1-\alpha)k}\right) < \frac{2c\alpha(\sqrt{2\alpha+(1-k)(1-\alpha)} - \sqrt{2\alpha})^2}{(1-k)(1-\alpha)^2} = \pi(p_d^*).$$

Table A1 summarizes the results in Cases 1, 2, 3.

Special case $k = 0$. In this case, CS defaults to CE model and $\tilde{\alpha}(k) = 0$. The last four cases in Table A1 do not exist. The rest of the above proof applies to CE by setting $k = 0$, and the results are captured in Proposition 1. □

Lemma A2. *Define function* $g(\alpha) = \frac{1}{16(1-\alpha)} - \frac{2\alpha}{\left(\sqrt{1+\alpha}+\sqrt{2\alpha}\right)^2}$. *Then, on the interval* $(0, 13-4\sqrt{10})$, *equation* $g(\alpha) = 0$ *has a unique solution* $\underline{\alpha} \approx 0.065$. *Furthermore,* $g(\alpha) > 0$ *for* $\alpha \in (0, \underline{\alpha})$ *and* $g(\alpha) < 0$ *for* $\alpha \in (\underline{\alpha}, 13 - 4\sqrt{10})$.

Proof. We are going to consider 3 cases:

(i) $\alpha \in \left(0, \frac{1}{32}\right)$. Then $g(\alpha) = \frac{\sqrt{1+\alpha}(1-32\alpha)+\sqrt{2\alpha}(1+32\alpha)}{16(1-\alpha)(\sqrt{1+\alpha}+\sqrt{2\alpha})} > 0$.

(ii) $\alpha \in \left[\frac{1}{32}, \frac{1}{4}\right]$. We can rewrite $g(\alpha)$ as

$$g(\alpha) = \frac{\left(\sqrt{1+\alpha}+\sqrt{2\alpha}\right)^2 - 32\alpha(1-\alpha)}{16(1-\alpha)\left(\sqrt{1+\alpha}+\sqrt{2\alpha}\right)^2}. \tag{A.19}$$

We define $z(\alpha) = \left(\sqrt{1+\alpha}+\sqrt{2\alpha}\right)^2 - 32\alpha(1-\alpha)$. For $\alpha \in \left[\frac{1}{32}, \frac{1}{4}\right]$, we have $sign\{g(\alpha)\} = sign\{z(\alpha)\}$, and g and z have the same roots. We are going to prove that $z(\alpha)$ has a unique root $\underline{\alpha}$ in the interval $\left[\frac{1}{32}, \frac{1}{4}\right]$. We have:

$$\frac{\partial z}{\partial \alpha}(\alpha) = 2\left(\sqrt{\frac{1+\alpha}{2\alpha}} + \frac{1}{2}\sqrt{\frac{2\alpha}{1+\alpha}}\right) - 29 + 64\alpha.$$

Over interval $\left[\frac{1}{32}, \frac{1}{4}\right]$, it can be shown that the function $y(\alpha) = \sqrt{\frac{1+\alpha}{2\alpha}} + \frac{1}{2}\sqrt{\frac{2\alpha}{1+\alpha}}$ is maximized when $\alpha = \frac{1}{32}$, in which case $y\left(\frac{1}{32}\right) \approx 4.185$. From (A.20), we see that:

$$\frac{\partial z}{\partial \alpha}(\alpha) \leq 2 \times 4.185 - 29 + 64 \times \frac{1}{4} < 0, \quad \forall\, \alpha \in \left[\frac{1}{32}, \frac{1}{4}\right]. \tag{A.20}$$

Moreover, $z\left(\frac{1}{32}\right) \approx 0.63$ and $z\left(\frac{1}{4}\right) \approx -2.67$. Therefore, it immediately follows that $z(\alpha) = 0$ has a unique solution $\underline{\alpha}$ in the interval $\left[\frac{1}{32}, \frac{1}{4}\right]$. Solving numerically the equation using Matlab we obtained $\underline{\alpha} \approx 0.065$. From (A.19) it immediately follows that $g(\underline{\alpha}) = 0$, $g(\alpha) > 0$ for $\alpha \in \left[\frac{1}{32}, \underline{\alpha}\right)$, and $g(\alpha) < 0$ when $\alpha \in \left(\underline{\alpha}, \frac{1}{4}\right]$.

(iii) $\alpha \in \left(\frac{1}{4}, 13 - 4\sqrt{10}\right)$. Here, $\frac{1}{16(1-\alpha)} < \frac{1}{8} < \frac{\alpha}{2} < \frac{2\alpha}{\left(\sqrt{1+\alpha}+\sqrt{2\alpha}\right)^2}$. Thus, $g(\alpha) < 0$. □

Proof of Proposition 5. For any $k \in [0, 1)$, we have:

$$13 - 4\sqrt{10} \leq \frac{13 + 2k + k^2 - 4\sqrt{10 + 2k^2 + 4k}}{(1-k)^2} \leq \frac{1}{\sqrt{3+k^2} + 1 - k}. \tag{A.21}$$

We split the analysis into several cases:

Case 1. $13 - 4\sqrt{10} \leq \alpha$**.** Consider a fixed $k \in [0, 1)$. We have several subcases:

(i) $\frac{13+2k+k^2-4\sqrt{10+2k^2+4k}}{(1-k)^2} < \alpha$. Then, $\pi^*_{CS} = \frac{c\alpha(1-k)}{2} \le \frac{c\alpha}{2} = \pi^*_{CE}$.

(ii) $13 - 4\sqrt{10} \le \alpha \le \frac{13+2k+k^2-4\sqrt{10+2k^2+4k}}{(1-k)^2}$. It can be shown that:

$$\pi^*_{CS} = \frac{2c\alpha(1-k)}{\left(\sqrt{2\alpha+(1-\alpha)(1-k)}+\sqrt{2\alpha}\right)^2} \le \frac{2c\alpha}{\left(\sqrt{1+\alpha}+\sqrt{2\alpha}\right)^2} \le \frac{c\alpha}{2} = \pi^*_{CE}.$$

Reconciling cases (i) and (ii) we see that $\pi^*_{CS} \le \pi^*_{CE}$ for any $k \in [0,1)$. Since the upper bound is attainable when $k = 0$, it follows that $k^*_{CS} = 0,\ \forall\ \alpha \ge 13 - 4\sqrt{10}$.

Case 2. $\underline{\alpha} \le \alpha < 13 - 4\sqrt{10}$. In this case, for any $k \in [0,1)$, it can be shown that:

$$\frac{(1-k)c[\alpha+(1-\alpha)k]}{4} \le \frac{c}{16(1-\alpha)} \le \frac{2\alpha}{\left(\sqrt{1+\alpha}+\sqrt{2\alpha}\right)^2} = \pi^*_{CE}.$$

Furthermore:

$$\frac{2c\alpha(1-k)}{\left(\sqrt{2\alpha+(1-\alpha)(1-k)}+\sqrt{2\alpha}\right)^2} \le \frac{2c\alpha}{\left(\sqrt{1+\alpha}+\sqrt{2\alpha}\right)^2} = \pi^*_{CE}.$$

From Lemma 4, regardless of where $\tilde{\alpha}(k)$ falls, we have $\pi^*_{CS} \le \pi^*_{CE}$. Since the upper bound is attainable when $k = 0$, we have $k^*_{CS} = 0,\ \forall\ \alpha \in \left[\underline{\alpha}, 13 - 4\sqrt{10}\right)$.

Case 3. $0 < \alpha < \underline{\alpha}$.In this case, for any $k \in [0,1)$, it can be shown that:

$$\frac{2c\alpha(1-k)}{\left(\sqrt{2\alpha+(1-\alpha)(1-k)}+\sqrt{2\alpha}\right)^2} \le \frac{2c\alpha}{\left(\sqrt{1+\alpha}+\sqrt{2\alpha}\right)^2} = \pi^*_{CE} < \frac{c}{16(1-\alpha)}. \quad \text{(A.22)}$$

We are going to show that the upper bound $\frac{c}{16(1-\alpha)}$ is attainable. Clearly, $\frac{(1-k)c[\alpha+(1-\alpha)k]}{4}$ is maximized at $k = k^*_{SS} = \frac{1-2\alpha}{2-2\alpha}$ and:

$$\left.\frac{(1-k)c[\alpha+(1-\alpha)k]}{4}\right|_{k=k^*_{SS}=\frac{1-2\alpha}{2-2\alpha}} = \frac{c}{16(1-\alpha)}.$$

Thus, for any $k \in [0,1)$:

$$\pi^*_{CS} \le \frac{c}{16(1-\alpha)} = \pi^*_{SS}.$$

In order to verify that the upper bound is attained by setting $k^*_{CS} = k^*_{SS} = \frac{1-2\alpha}{2-2\alpha}$, all that is left to check is that $\tilde{\alpha}(k^*_{SS}) > \underline{\alpha}$ due to Lemma 4. Pugging $k = \frac{1-2\alpha}{2-2\alpha}$ in $h_k(\alpha)$, we obtain:

$$h_{k^*_{SS}}(\alpha) = \frac{1}{2}\left(\sqrt{2\alpha+\frac{1}{2}}+\sqrt{2\alpha}\right)^2 - 8\alpha. \quad \text{(A.23)}$$

The unique solution (implicitly equal to $\tilde{\alpha}(k^*_{SS})$) to equation $h_{k^*_{SS}}(\alpha) = 0$ over the interval

$(0,\infty)$ is $\tilde{\alpha}(k^*_{SS}) = \frac{2+\sqrt{2}}{32} \approx 0.107 > \underline{\alpha} \approx 0.065$. Therefore, it immediately follows that:

$$k^*_{CS} = k^*_{SS} = \frac{1-2\alpha}{2-2\alpha}, \quad p^*_{CS} = p^*_{SS}, \quad \pi^*_{CS} = \pi^*_{SS}, \quad SW^*_{CS} = SW^*_{SS},$$

and adoption occurs solely in period 2. □

Proof of Corollary 1. Follows immediately from Propositions 1, 4, and 5. □

Proof of Corollary 2. Follows immediately from Propositions 3, 4, and 5. □

Lemma A3. *Define* $f(x) = \frac{2x}{(\sqrt{1+x}+\sqrt{2x})^2}$. *Then,* $0 \le f(x) < \frac{1}{4}$, $\forall\ x \in [0, 13-4\sqrt{10}]$.

Proof. We have $f(x) < \frac{1}{4} \iff 5x - 1 < 2\sqrt{(1+x)2x}$. For $x \in [0, 0.2]$ this is obvious. Otherwise: $5x - 1 < 2\sqrt{(1+x)2x} \iff 17x^2 - 18x + 1 < 0,\ \forall\ x \in (0.2, 13-4\sqrt{10}]$. The two roots to equation $17x^2 - 18x + 1 = 0$ are $x_1 = \frac{1}{17} < 0.2$ and $x_2 = 1$. Therefore, $17x^2 - 18x + 1 < 0$ for all $x \in (0.2, 13-4\sqrt{10}] \subset \left[\frac{1}{17}, 1\right]$. □

Lemma A4. *Define* $\delta = \alpha_b + \delta_b - \alpha = \frac{\alpha_b - \alpha_a}{\gamma+1} + \delta_b$. *When the firm can choose among CE, FLF, and TLF, its optimal strategy is as follows:*

(a) if $0 \le \alpha, \alpha+\delta < 13-4\sqrt{10}$ *and*

- *(i) if* $(1+\gamma)f(\alpha) \le \gamma f(\alpha+\delta)$, *then* $\pi^*_{CE} \le \pi^*_{FLF} \le \pi^*_{TLF}$.
- *(ii) if* $(1+\gamma)f(\alpha) > \gamma f(\alpha+\delta)$, *then* $\pi^*_{FLF} \le \pi^*_{CE} \le \pi^*_{TLF}$.

(b) if $0 \le \alpha < 13-4\sqrt{10} \le \alpha+\delta$, *then, since* $f(\alpha) < \frac{1}{4}$, *we have:*

- *(i) if* $(1+\gamma)f(\alpha) \le \frac{\gamma(\alpha+\delta)}{2} < \frac{1+\gamma}{4}$, *then* $\pi^*_{CE} \le \pi^*_{FLF} \le \pi^*_{TLF}$.
- *(ii) if* $(1+\gamma)f(\alpha) < \frac{1+\gamma}{4} \le \frac{\gamma(\alpha+\delta)}{2}$, *then* $\pi^*_{CE} \le \pi^*_{TLF} \le \pi^*_{FLF}$.
- *(iii) if* $\frac{\gamma(\alpha+\delta)}{2} \le (1+\gamma)f(\alpha) < \frac{1+\gamma}{4}$, *then* $\pi^*_{FLF} \le \pi^*_{CE} \le \pi^*_{TLF}$.

(c) if $0 \le \alpha+\delta < 13-4\sqrt{10} \le \alpha$, *then* $f(\alpha+\delta) < \frac{1}{4}$ *and:*

- *(i) if* $\gamma f(\alpha+\delta) \le \frac{\alpha(1+\gamma)}{2} < \frac{1+\gamma}{4}$, *then* $\pi^*_{FLF} \le \pi^*_{CE} \le \pi^*_{TLF}$.
- *(ii) if* $\gamma f(\alpha+\delta) < \frac{1+\gamma}{4} \le \frac{\alpha(1+\gamma)}{2}$, *then* $\pi^*_{FLF} \le \pi^*_{TLF} \le \pi^*_{CE}$.
- *(iii) if* $\frac{\alpha(1+\gamma)}{2} \le \gamma f(\alpha+\delta) < \frac{1+\gamma}{4}$, *then* $\pi^*_{CE} \le \pi^*_{FLF} \le \pi^*_{TLF}$.

(d) if $13-4\sqrt{10} \le \alpha, \alpha+\delta$, *and*

- *(i)* $(1+\gamma)\alpha \le \gamma(\alpha+\delta) \le \frac{1+\gamma}{2}$, *then* $\pi^*_{CE} \le \pi^*_{FLF} \le \pi^*_{TLF}$.
- *(ii)* $\gamma(\alpha+\delta) \le (1+\gamma)\alpha \le \frac{1+\gamma}{2}$, *then* $\pi^*_{FLF} \le \pi^*_{CE} \le \pi^*_{TLF}$.
- *(iii)* $(1+\gamma)\alpha \le \frac{1+\gamma}{2} \le \gamma(\alpha+\delta)$, *then* $\pi^*_{CE} \le \pi^*_{TLF} \le \pi^*_{FLF}$.
- *(iv)* $\gamma(\alpha+\delta) \le \frac{1+\gamma}{2} \le (1+\gamma)\alpha$, *then* $\pi^*_{FLF} \le \pi^*_{TLF} \le \pi^*_{CE}$.
- *(v)* $\frac{1+\gamma}{2} \le (1+\gamma)\alpha \le \gamma(\alpha+\delta)$, *then* $\pi^*_{TLF} \le \pi^*_{CE} \le \pi^*_{FLF}$.

(vi) $\frac{1+\gamma}{2} \le \gamma(\alpha+\delta) \le (1+\gamma)\alpha$*, then* $\pi^*_{TLF} \le \pi^*_{FLF} \le \pi^*_{CE}$.

Proof. Follows directly from Propositions 1, 2, 3, Lemma 2, and Lemma A3. □

Lemma A5. *Define* δ *as in Lemma A4. From a profit perspective, the following hold:*

(a) If $\alpha \ge \max\left\{\frac{1}{2}, \gamma\delta\right\}$*, then* CE *is the dominating strategy.*

(b) If $\alpha+\delta \ge \frac{1}{2}+\frac{1}{2\gamma}$ *and* $\gamma\delta \ge \alpha$*, then* FLF *is the dominating strategy.*

(c) If $\alpha \le \frac{1}{2}$ *and* $\alpha+\delta \le \frac{1}{2}+\frac{1}{2\gamma}$*, then* TLF *is the dominating strategy.*

(d) Strategies CE*,* FLF*, and* TLF *are equivalent when* $\alpha = \frac{1}{2}$ *and* $\delta = \frac{1}{2\gamma}$.

Proof. SS and CS are either dominated by CE or TLF, as shown in Corollaries 1 and 2. The results follow directly by consolidating the separate cases in Lemma A4. □

Proof of Theorem 1. Theorem 1 is restating the results in Lemma A5 using the following transformations (i) $\alpha+\delta = \alpha_b+\delta_b$, and (ii) $\alpha > \gamma\delta \iff \frac{\alpha_a}{\gamma} > \delta_b$. Note that, when $\alpha_a > \frac{\gamma+1}{2}$, then $\alpha > \frac{1}{2}$. When $\alpha_a \le \frac{\gamma+1}{2}$, then $\alpha \le \frac{1}{2}$ if and only if $\alpha_b \le \frac{1}{\gamma}\left(\frac{\gamma+1}{2}-\alpha_a\right)$. □

Proof of Corollary 3. Follows immediately from Propositions 1, 2, and 3. □

Lemma A6. *Define* $\phi(x) = 1 - \frac{1+2x+2x^2}{2(1+x)\left(\sqrt{1+x}+\sqrt{2x}\right)^2}$*. Then,* $\frac{\partial\phi}{\partial x} > 0,\ \forall\ x \in [0, 13-4\sqrt{10}]$.

Proof. Denote $\phi_1(x) = 1+2x+2x^2$ and $\phi_2(x) = 2(1+x)\left(\sqrt{1+x}+\sqrt{2x}\right)^2$. Then, we have $sign\frac{\partial\phi}{\partial x} = -sign(\phi_1'\phi_2 - \phi_1\phi_2')$. After simplifying, we obtain:

$$\begin{aligned}\phi_1'\phi_2 - \phi_1\phi_2' \quad = \quad & (2+8x+4x^2)\left(\sqrt{1+x}+\sqrt{2x}\right)^2 \\ & -(2+4x+4x^2)\left(\sqrt{1+x}+\sqrt{2x}\right)\left(\sqrt{1+x}+\sqrt{\frac{2(1+x)^2}{x}}\right). \end{aligned} \tag{A.24}$$

For any $x \in (0, 13-4\sqrt{10}]$, it can be shown that $\sqrt{\frac{2(1+x)^2}{x}} \ge 3 > 2\sqrt{2x}+\sqrt{1+x}$. Thus,

$$\sqrt{1+x}+\sqrt{\frac{2(1+x)^2}{x}} > 2(\sqrt{2x}+\sqrt{1+x}). \tag{A.25}$$

Using (A.24) and (A.25), we obtain $\phi_1'\phi_2 - \phi_1\phi_2' < -(2+4x^2)\left(\sqrt{1+x}+\sqrt{2x}\right)^2 < 0$. Thus, $\frac{\partial\phi}{\partial x} > 0$ for all $x \in [0, 13-4\sqrt{10}]$. □

Proof of Theorem 2. (a) When $\alpha < 13-4\sqrt{10}$, then $SW^*_{CE} = c\phi(\alpha) < c\phi(13-4\sqrt{10}) \approx c \cdot 0.82 < \frac{7c}{8} = SW^*_{TLF}$, due to Lemma A6. When $\alpha \ge 13-4\sqrt{10}$, then $SW^*_{CE} = \frac{3c}{4} < \frac{7c}{8} = SW^*_{TLF}$. Therefore $SW^*_{CE} < SW^*_{TLF}$ for any $\alpha > 0$. By direct comparison it can be also shown that $SW^*_{SS} < SW^*_{TLF}$ for any $\alpha > 0$. When $\alpha \ge \underline{\alpha}$, $SW^*_{CS} = SW^*_{CE} < SW^*_{TLF}$. When $\alpha < \underline{\alpha}$, then $SW^*_{CS} = SW^*_{SS} < SW^*_{TLF}$. Thus, $SW^*_{CS} < SW^*_{TLF}$ for any $\alpha > 0$.

(b) Case 1. $0 < \alpha_b + \delta_b < 13 - 4\sqrt{10}$. Then:

$$SW^*_{FLF} \geq SW^*_{TLF} \iff \phi(\alpha_b + \delta_b) \geq \frac{7\gamma - 1}{8\gamma}.$$

We know from Lemma A6 that $\phi(\cdot)$ is strictly increasing over the interval $[0, 13 - 4\sqrt{10}]$ with $\phi(0) = \frac{1}{2}$. We discuss several subcases based on the value of γ.

- Case 1.a: $\gamma \leq \frac{1}{3}$. Then $\frac{7\gamma-1}{8\gamma} \leq \frac{1}{2} < \phi(\alpha_b + \delta_b)$ for all values $\alpha_b + \delta_b > 0$. Therefore, we have $SW^*_{FLF} \geq SW^*_{TLF}$.
- Case 1.b: $\gamma \geq \frac{1}{7-8\phi(13-4\sqrt{10})} \approx 2.262$. Then $\frac{7\gamma-1}{8\gamma} \geq \phi(13 - 4\sqrt{10}) > \phi(\alpha_b + \delta_b)$. Therefore, we have $SW^*_{FLF} < SW^*_{TLF}$.
- Case 1.c: $\frac{1}{3} < \gamma < \frac{1}{7-8\phi(13-4\sqrt{10})}$. Then $\frac{1}{2} < \frac{7\gamma-1}{8\gamma} \leq 13 - 4\sqrt{10}$. In this case, $\phi^{-1}\left(\frac{7\gamma-1}{8\gamma}\right)$ is properly defined and belongs to the interval $[0, 13 - 4\sqrt{10}]$. Consequently, we have two possible outcomes:
  - if $0 < \alpha_b + \delta_b < \phi^{-1}\left(\frac{7\gamma-1}{8\gamma}\right)$, then $SW^*_{FLF} < SW^*_{TLF}$.
  - if $\phi^{-1}\left(\frac{7\gamma-1}{8\gamma}\right) \leq \alpha_b + \delta_b < 13 - 4\sqrt{10}$, then $SW^*_{FLF} \geq SW^*_{TLF}$.

Case 2. $\alpha_b + \delta_b \geq 13 - 4\sqrt{10}$. Then:

$$SW^*_{FLF} \geq SW^*_{TLF} \iff \frac{c(4+3\gamma)}{4(1+\gamma)} \geq \frac{7c}{8} \iff \gamma \leq 1. \quad \square$$

Sketch of the derivation of optimal profits for each model when $w \to \infty$ and $\max\{\alpha, \alpha_b + \delta_b\} < \frac{1}{4}$.

(i) *CE* model. Regardless of how strong network effects are, since nothing is given for free, adoption cannot start unless $p < 2c\alpha$. In that case, we have $\theta_1 = \frac{p}{2c\alpha}$. However, due to very strong word-of-mouth effects, consumers immediately update their priors to the correct value at the beginning of period 1, i.e. $c_2 = c$. Thus, $\theta_2 = \frac{p}{c}$. Therefore, adoption in period 2 occurs if and only if $\alpha < \frac{1}{2}$. The firm's optimization problem is:

$$\max_{0<p<2c\alpha} p\left(1 - \min\left\{\frac{p}{2c\alpha}, \frac{p}{c}\right\}\right).$$

Then, when consumers strongly underestimate the value of the product ($\alpha \leq \frac{1}{4}$), it easily follows that $p^*_{CE} \approx 2c\alpha$ because the upper bound constraint on price becomes active. Consequently:

$$\lim_{w\to\infty} \pi^*_{CE}(\alpha) \approx 2c\alpha(1 - 2\alpha) < \frac{c}{4}, \quad \forall \quad \alpha < \frac{1}{4}.$$

(ii) *FLF* model. Similar to the *CE* model, replacing α with $\alpha_b + \delta_b$.

(iii) <u>TLF model.</u> Similar to the case when $w = 1$.

(iv) <u>SS model.</u> When $w \to \infty$ and $\alpha < \frac{1}{4}$, negligible positive seeding is beneficial since it raises consumers' willingness to pay at a relatively negligible cost. Therefore, cannibalizing a negligible portion of the adoption via seeding, the firm reaches period 2 with consumers fully informed about the value of the product. It can be formally shown that:

$$\lim_{w\to\infty} \pi^*_{SS}(\alpha) = \frac{c}{4} \quad \text{and} \lim_{w\to\infty} k^*_{SS}(\alpha) = 0^+, \quad \forall \quad \alpha < \frac{1}{4}.$$

(v) <u>CS model.</u> Similar to the SS case, $w \to \infty$ and $\alpha < \frac{1}{4}$, negligible positive seeding is beneficial since it raises consumers' willingness to pay at a relatively negligible cost. If the firm prices at $p < 2\alpha c$, then it obtains the same profit as in the CE case (since in the latter also consumers are fully informed about the value of the product in period 2). However, unlike in the CE case, in the CS case the firm can price also in the interval $[2\alpha c$, c) and have adoption only in period 2 since word-of-mouth effects are initiated by seeding. It immediately follows that, when $\alpha < \frac{1}{4}$, then $p^* = \frac{c}{2}$ and

$$\lim_{w\to\infty} \pi^*_{CS}(\alpha) = \frac{c}{4} \quad \text{and} \lim_{w\to\infty} k^*_{CS}(\alpha) = 0^+, \quad \forall \quad \alpha < \frac{1}{4}. \quad \square$$